MAX PLANCK INSTITUTE
FOR EXTRATERRESTRIAL PHYSICS

MOLECULAR SPECTROSCOPY FOR ASTROCHEMISTRY

Part I — Context and Theory

Valerio Lattanzi — CAS@MPE

MAIN REFERENCES AND CREDITS:

- Master in Astrochemistry — Ewine van Dishoeck (University of Leiden, 2010)
- 2021 Census of Interstellar, Circumstellar, Extragalactic, Protoplanetary Disk, and Exoplanetary Molecules — Brett McGuire (ApJS 2022)
- Interstellar Medium: Physics and Chemistry — Javier Goicoechea (IRAM 2021 summer school)

- Microwave Molecular Spectra — W. Gordy and R.L. Cook (John Whiley and Sons, Inc, 1984)
- Spectra of Atoms and Molecules — P.F. Bernath (Oxford University Press, 2005)
- Molecular Rotation Spectra — H.W. Kroto (Dover Publications Inc. 1992)

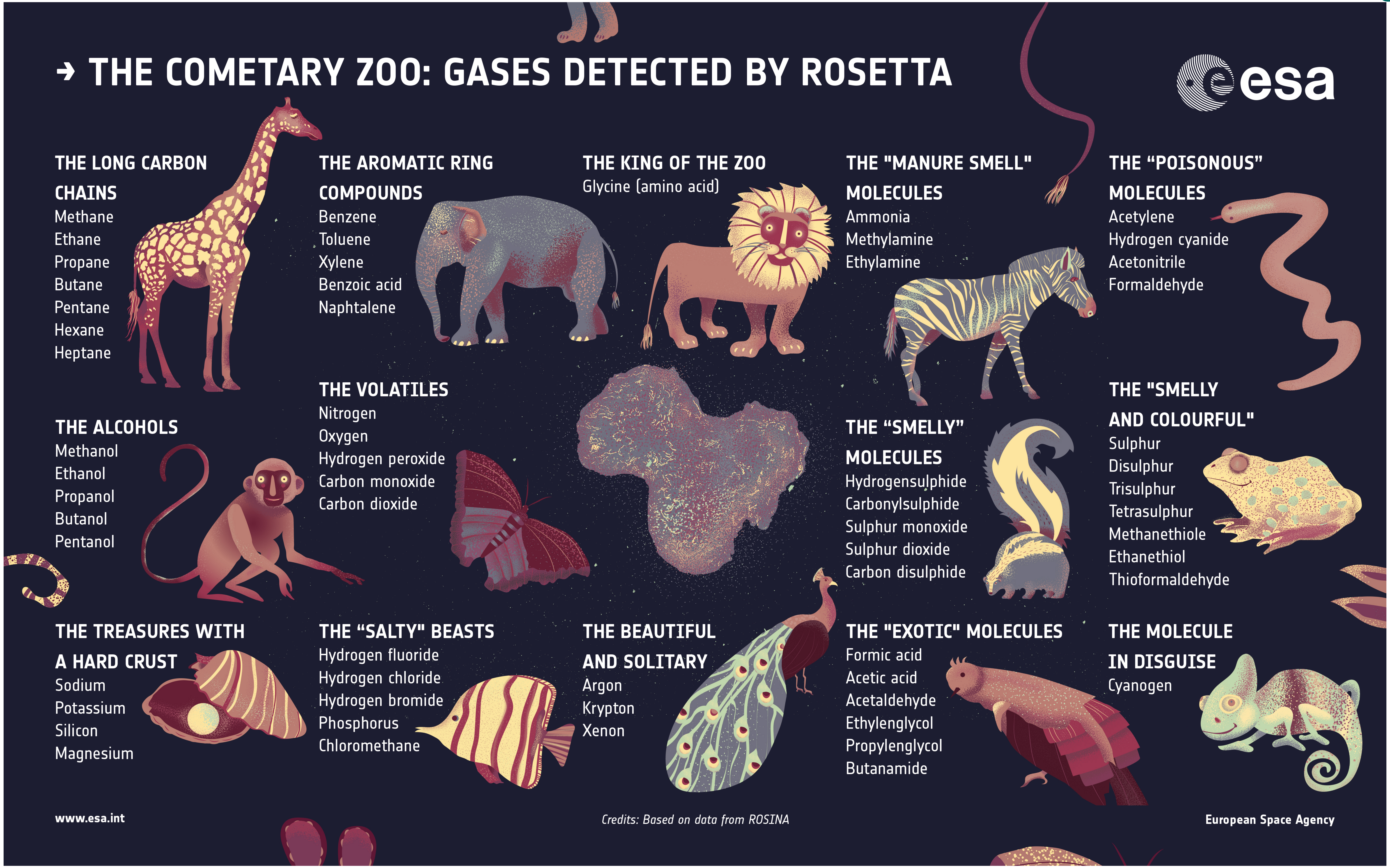
MENU OF THE DAY

- Molecules in the ISM.
- Why? Where? What?
- How to derive useful information?

- Molecular (rotational) spectroscopy
- A bit of theory…

MOLECULES ARE EVERYWHERE!

INCLUDING COMETS: COMET 67P/C-G



A BIT OF PHYSICS

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SLOW chemistry!

and astrophysicists did not expect many molecules in space...

SCIENTISTS WERE SKEPTICAL…

It is difficult to admit the existence of molecules in interstellar space because when once a molecule becomes dissociated there seems no chance of the atoms joining up again.[28]

A. Eddington (1926)

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G. Herzberg (1950)

MOLECULAR SPECTRA

and

MOLECULAR STRUCTURE

I. SPECTRA OF DIATOMIC MOLECULES

BY

GERHARD HERZBERG, F.R.S.
National Research Council of Canada

With the co-operation, in the first edition, of
J. W. T. SPINKS, F.R.S.C.

The observation that in interstellar space only the very lowest rotational levels of CH, CH^+, and CN are populated is readily explained by the depopulation of the higher levels by emission of the far infrared rotation spectrum (see p. 4 .) and by the lack of excitation to these levels by collisions or radiation. The intensity of the rotation spectrum of CN is much smaller than that of CH or CH^+ on account of the smaller dipole moment as well as the smaller frequency [due to the factor ν^4 in (I, 48)]. That is why lines from the second lowest level ($K = 1$) have been observed for CN. From the intensity ratio of the lines with $K = 0$ and $K = 1$ a rotational temperature of 2.3° K follows, which has of course only a very restricted meaning.

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In diffuse clouds collisional excitation is negligible

T_{rot} *(molecules)* $\approx T_{bg} \approx T_{CMB}$

KNOWN INTERSTELLAR MOLECULES

2 Atoms
- CH
- CN
- CH^+
- OH
- CO
- H_2
- SiO
- CS
- SO
- SiS
- NS
- C_2
- NO
- HCl
- $NaCl$
- $AlCl$
- KCl
- AlF
- PN
- SiC
- CP
- NH
- SiN
- SO^+
- CO^+
- HF
- N_2
- CF^+
- PO
- O_2
- AlO
- CN^-
- OH^+
- SH^+
- HCl^+
- SH
- TiO
- ArH^+
- NS^+
- HeH^+
- VO
- PO^+

3 Atoms
- H_2O
- HCO^+
- HCN
- OCS
- HNC
- H_2S
- N_2H^+
- C_2H
- SO_2
- HCO
- HNO
- HCS^+
- HOC^+
- SiC_2
- C_2S
- C_3
- CO_2
- CH_2
- C_2O
- $MgNC$
- NH_2
- $NaCN$
- N_2O
- $MgCN$
- H_3^+
- $SiCN$
- $AlNC$
- $SiNC$
- HCP
- CCP
- $AlOH$
- H_2O^+
- H_2Cl^+
- KCN
- $FeCN$
- HO_2
- TiO_2
- CCN
- $SiCSi$
- S_2H
- HCS
- HSC
- NCO
- $CaNC$
- NCS

4 Atoms
- NH_3
- H_2CO
- $HNCO$
- H_2CS
- C_2H_2
- C_3N
- $HNCS$
- $HOCO^+$
- C_3O
- *l*-C_3H
- $HCNH^+$
- H_3O^+
- C_3S
- *c*-C_3H
- HC_2N
- H_2CN
- SiC_3
- CH_3
- C_3N^-
- PH_3
- $HCNO$
- $HOCN$
- $HSCN$
- $HOOH$
- *l*-C_3H^+
- $HMgNC$
- $HCCO$
- $CNCN$
- $HONO$
- $MgCCH$
- $HCCS$
- $HNCN$
- H_2NC
- $HCCS^+$

5 Atoms
- HC_3N
- $HCOOH$
- CH_2NH
- NH_2CN
- H_2CCO
- C_4H
- SiH_4
- *c*-C_3H_2
- CH_2CN
- C_5
- SiC_4
- H_2CCC
- CH_4
- $HCCNC$
- $HNCCC$
- H_2COH^+
- C_4H^-
- $CNCHO$
- $HNCNH$
- CH_3O
- NH_3D^+
- H_2NCO^+
- $NCCNH^+$
- CH_3Cl
- MgC_3N
- HC_3O^+
- NH_2OH
- HC_3S^+
- H_2CCS
- C_4S
- $CHOSH$
- $HCSCN$
- HC_3O

6 Atoms
- CH_3OH
- CH_3CN
- NH_2CHO
- CH_3SH
- C_2H_4
- C_5H
- CH_3NC
- HC_2CHO
- H_2C_4
- C_5S
- HC_3NH^+
- C_5N
- HC_4H
- HC_4N
- *c*-H_2C_3O
- CH_2CNH
- C_5N^-
- $HNCHCN$
- SiH_3CN
- MgC_4H
- CH_3CO^+
- H_2CCCS
- CH_2CCH
- $HCSCCH$
- C_5O
- C_5H^+
- *c*-C_5H

7 Atoms
- CH_3CHO
- CH_3CCH
- CH_3NH_2
- CH_2CHCN
- HC_5N
- C_6H
- *c*-C_2H_4O
- CH_2CHOH
- C_6H^-
- CH_3NCO
- HC_5O
- $HOCH_2CN$
- HC_4NC
- HC_3HNH
- *c*-C_3HCCH
- MgC_5N
- CH_2C_3N

12 Atoms
- C_6H_6
- *n*-C_3H_7CN
- *i*-C_3H_7CN
- 1-C_5H_5CN
- 2-C_5H_5CN
- *n*-$CH_3CH_2CH_2OH$
- *i*-$CH_3CH_2CH_2OH$

8 Atoms
- $HCOOCH_3$
- CH_3C_3N
- C_7H
- CH_3COOH
- H_2C_6
- CH_2OHCHO
- HC_6H
- CH_2CHCHO
- CH_2CCHCN
- NH_2CH_2CN
- CH_3CHNH
- CH_3SiH_3
- NH_2CONH_2
- $HCCCH_2CN$
- CH_2CHCCH
- MgC_6H
- $C_2H_3NH_2$
- $HOCHCHOH$

9 Atoms
- CH_3OCH_3
- CH_3CH_2OH
- CH_3CH_2CN
- HC_7N
- CH_3C_4H
- C_8H
- CH_3CONH_2
- C_8H^-
- CH_2CHCH_3
- CH_3CH_2SH
- HC_7O
- CH_3NHCHO
- $H_2CCCHCCH$
- $HCCCHCHCN$
- H_2CCHC_3N

10 Atoms
- CH_3COCH_3
- $HOCH_2CH_2OH$
- CH_3CH_2CHO
- CH_3C_5N
- CH_3CHCH_2O
- CH_3OCH_2OH
- C_6H_4
- C_2H_5NCO
- HC_7NH^+
- $CH_3CHCHCN$
- CH_2CCH_3CN
- CH_2CHCH_2CN

11 Atoms
- HC_9N
- CH_3C_6H
- C_2H_5OCHO
- CH_3COOCH_3
- CH_3COCH_2OH
- C_5H_6
- $NH_2CH_2CH_2OH$
- CH_2CCHC_4H

13+ Atoms
- C_6H_5CN
- $HC_{11}N$
- *c*-$C_5H_4CCH_2$
- 1-$C_{10}H_7CN$
- 2-$C_{10}H_7CN$
- C_9H_8
- C_{60}
- C_{60}^+
- C_{70}

267 Molecules

Last Updated: 15 Aug 2022

Created with **ASTROMOL** v2021.5.0
bmcguir2.github.io/astromol
McGuire 2022 *ApJS* 259, 30

ATOMIC REPRESENTATION OF ISM MOLECULES

1 **197** **H** 1.00794 Hydrogen																	2 **1** **He** 4.002602 Helium
3 **Li** 6.941 Lithium	4 **Be** 9.012182 Beryllium											5 **B** 10.811 Boron	6 **214** **C** 12.0107 Carbon	7 **105** **N** 14.0067 Nitrogen	8 **84** **O** 15.9994 Oxygen	9 **3** **F** 18.9984032 Fluorine	10 **Ne** 20.1797 Neon
11 **2** **Na** 22.98977 Sodium	12 **8** **Mg** 24.305 Magnesium											13 **5** **Al** 26.981538 Aluminum	14 **13** **Si** 28.0855 Silicon	15 **7** **P** 30.973761 Phosphorus	16 **33** **S** 32.065 Sulfur	17 **7** **Cl** 35.453 Chlorine	18 **1** **Ar** 39.948 Argon
19 **2** **K** 39.0983 Potassium	20 **1** **Ca** 40.078 Calcium	21 **Sc** 44.95591 Scandium	22 **2** **Ti** 47.867 Titanium	23 **1** **V** 50.9415 Vanadium	24 **Cr** 51.9961 Chromium	25 **Mn** 54.938049 Manganese	26 **1** **Fe** 55.845 Iron	27 **Co** 58.9332 Cobalt	28 **Ni** 58.6934 Nickel	29 **Cu** 63.546 Copper	30 **Zn** 65.409 Zinc	31 **Ga** 69.723 Gallium	32 **Ge** 72.64 Germanium	33 **As** 74.9216 Arsenic	34 **Se** 78.96 Selenium	35 **Br** 79.904 Bromine	36 **Kr** 83.798 Krypton
37 **Rb** 85.4678 Rubidium	38 **Sr** 87.62 Strontium	39 **Y** 88.90585 Yttrium	40 **Zr** 91.224 Zirconium	41 **Nb** 92.90638 Niobium	42 **Mo** 95.94 Molybdenum	43 **Tc** 98 Technetium	44 **Ru** 101.07 Ruthenium	45 **Rh** 102.9055 Rhodium	46 **Pd** 106.42 Palladium	47 **Ag** 107.8682 Silver	48 **Cd** 112.411 Cadmium	49 **In** 114.818 Indium	50 **Sn** 118.71 Tin	51 **Sb** 121.76 Antimony	52 **Te** 127.6 Tellurium	53 **I** 126.90447 Iodine	54 **Xe** 131.293 Xenon
55 **Cs** 132.90545 Cesium	56 **Ba** 137.327 Barium		72 **Hf** 178.49 Hafnium	73 **Ta** 180.9479 Tantalum	74 **W** 183.84 Tungsten	75 **Re** 186.207 Rhenium	76 **Os** 190.23 Osmium	77 **Ir** 192.217 Iridium	78 **Pt** 195.078 Platinum	79 **Au** 196.96655 Gold	80 **Hg** 200.59 Mercury	81 **Tl** 204.3833 Thallium	82 **Pb** 207.2 Lead	83 **Bi** 208.98038 Bismuth	84 **Po** 209 Polonium	85 **At** 210 Astatine	86 **Rn** 222 Radon
87 **Fr** 223 Francium	88 **Ra** 226 Radium		104 **Rf** 261 Rutherfordium	105 **Db** 262 Dubnium	106 **Sg** 266 Seaborgium	107 **Bh** 264 Bohrium	108 **Hs** 277 Hassium	109 **Mt** 268 Meitnerium	110 **Ds** 281 Darmstadtium	111 **Rg** 272 Roentgenium	112 **Cn** 285 Copernicium	113 **Nh** 286 Nihonium	114 **Fl** 289 Flerovium	115 **Mc** 289 Moscovium	116 **Lv** 293 Livermorium	117 **Ts** 294 Tennessine	118 **Og** 294 Oganesson

MOLECULES AS CHEMICAL AND PHYSICAL TOOLS

- **Identification** => chemical compositions, differentiation, time evolution

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- **Identification** => chemical compositions, differentiation, time evolution

- **Line strengths** => molecular abundances
- **Line ratios** => environment temperatures, densities
- **Line profiles** => kinematical analysis

MOLECULES AS TOOL FOR CHEMICAL DIFFERENTIATION

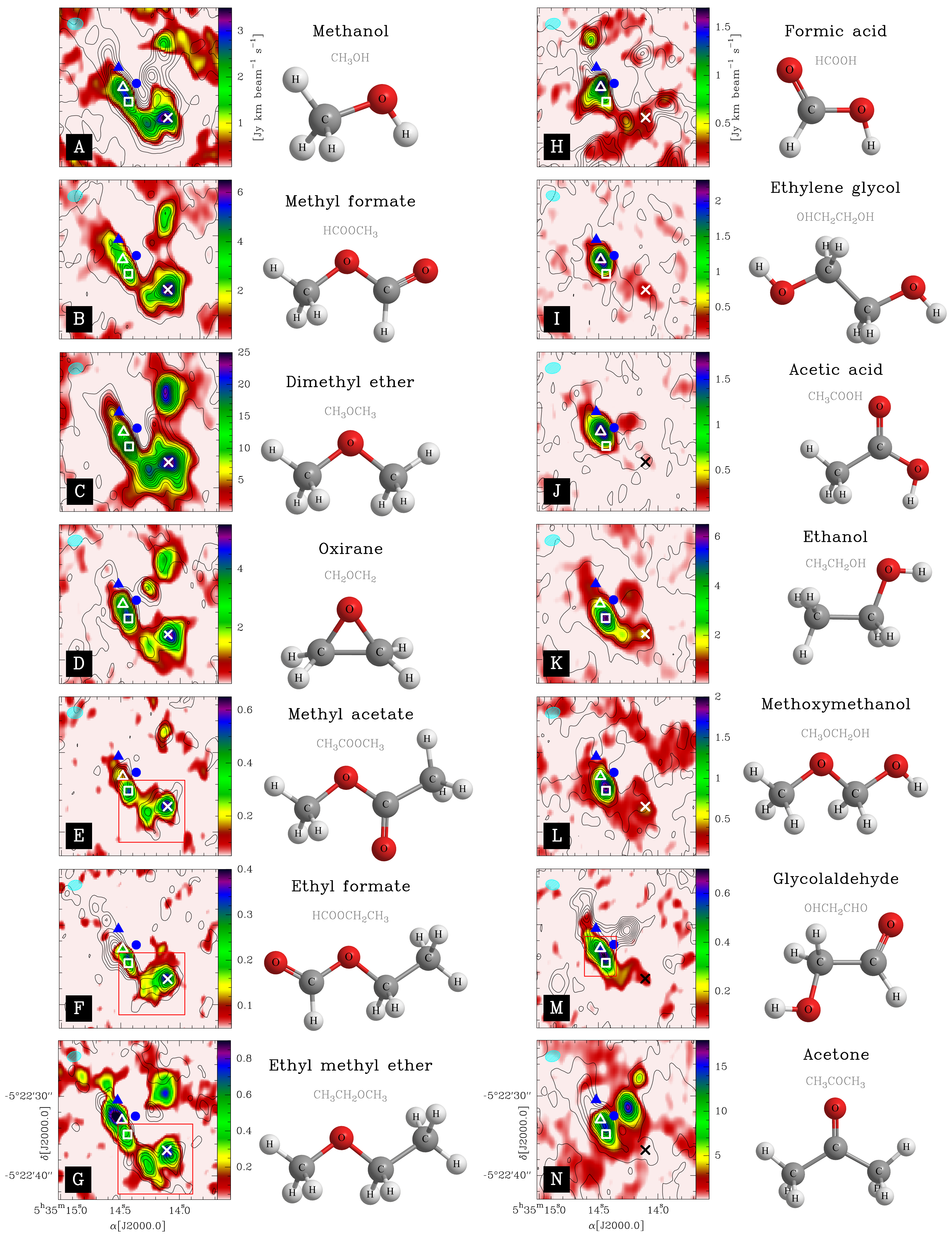
IRAM 30m view of L183 (prestellar core)

Continuum in black/white contour

Lattanzi+ in prep.

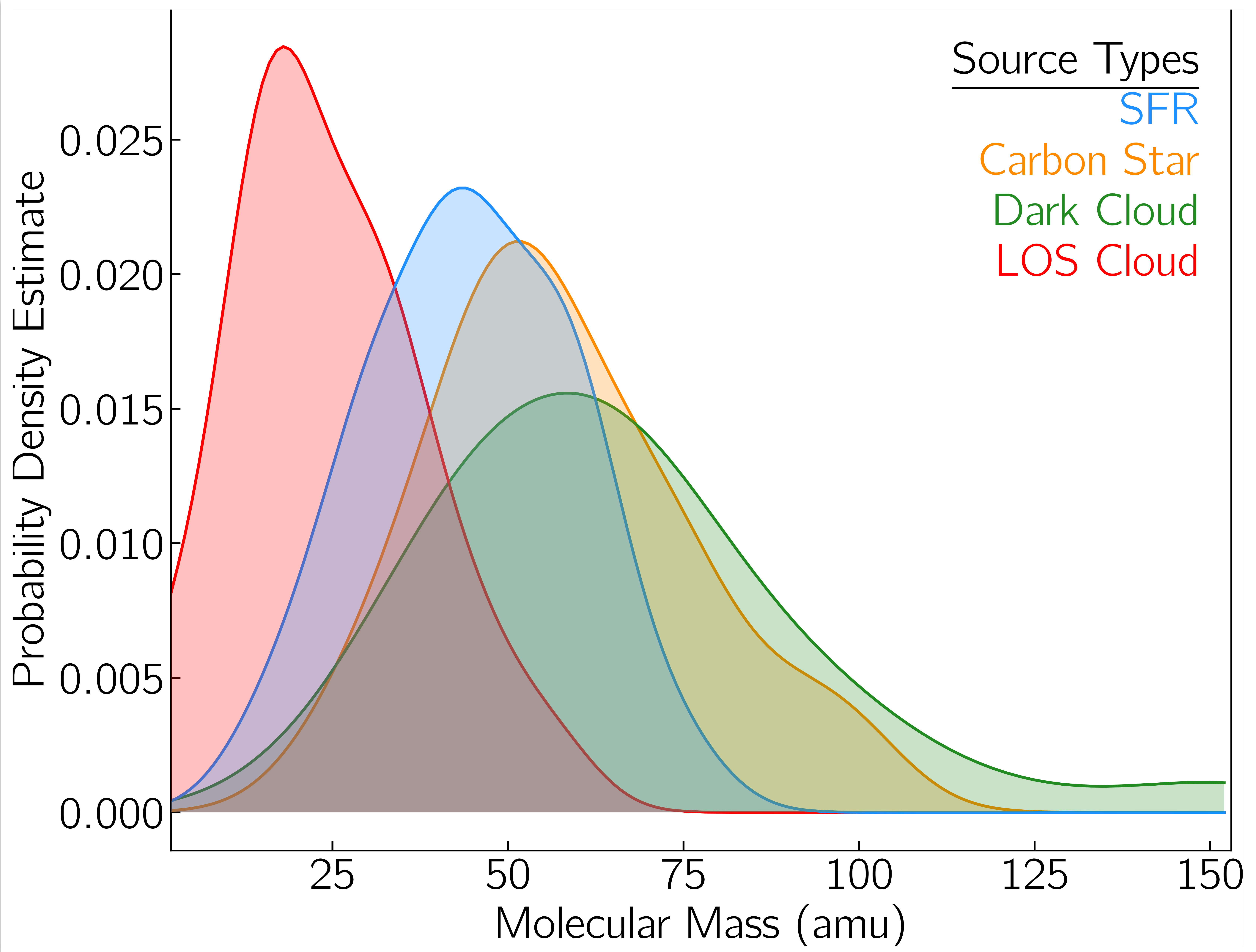
MOLECULES AS TOOL FOR CHEMICAL DIFFERENTIATION

ALMA SV view ORION KL (high-mass SFR)

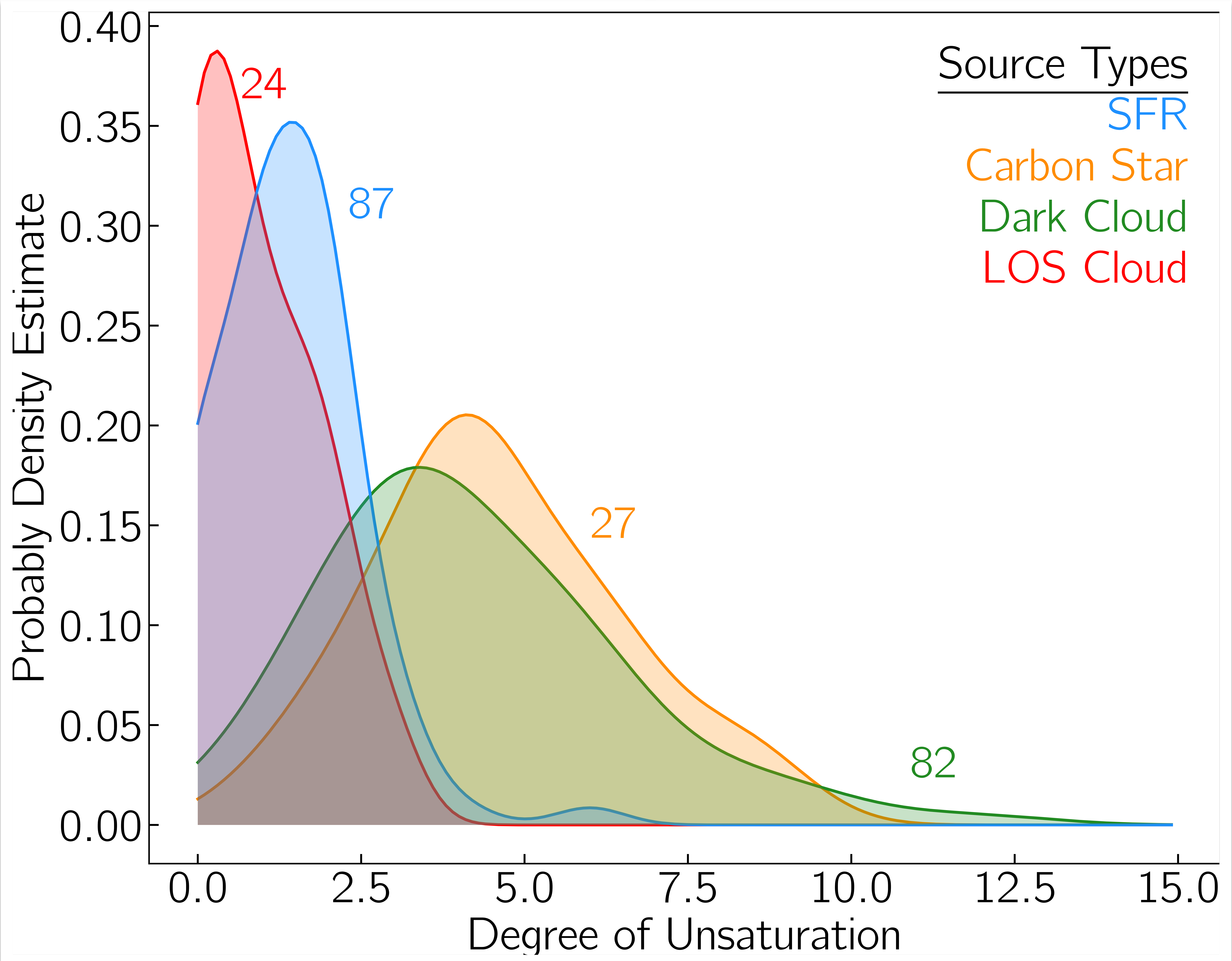


Colour and contours represent two different transitions

Tercero+ 2018



Source Types
SFR
Carbon Star
Dark Cloud
LOS Cloud
Probability Density Estimate
0.025
0.020
0.015
0.010
0.005
0.000
25
50
75
100
125
150
Molecular Mass (amu)



Source Types
SFR
Carbon Star
Dark Cloud
LOS Cloud
24
87
27
82
Probably Density Estimate
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
2.5
5.0
7.5
10.0
12.5
15.0
Degree of Unsaturation

All very nice, but first…

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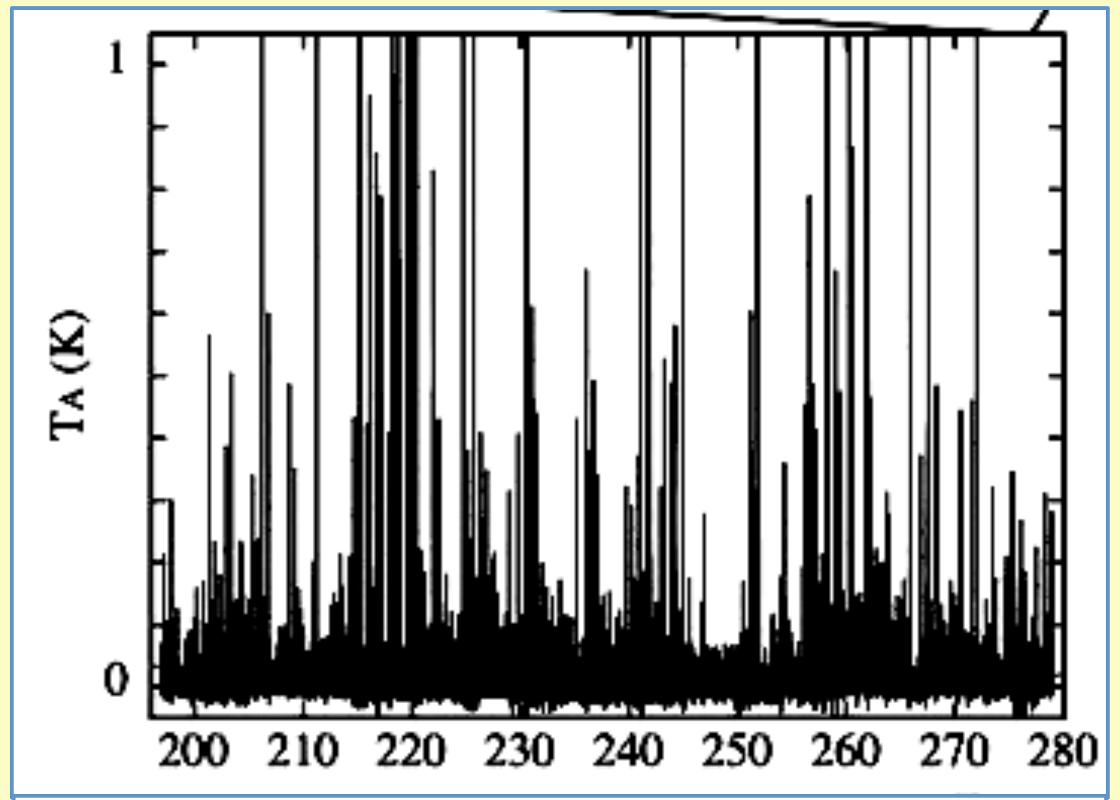
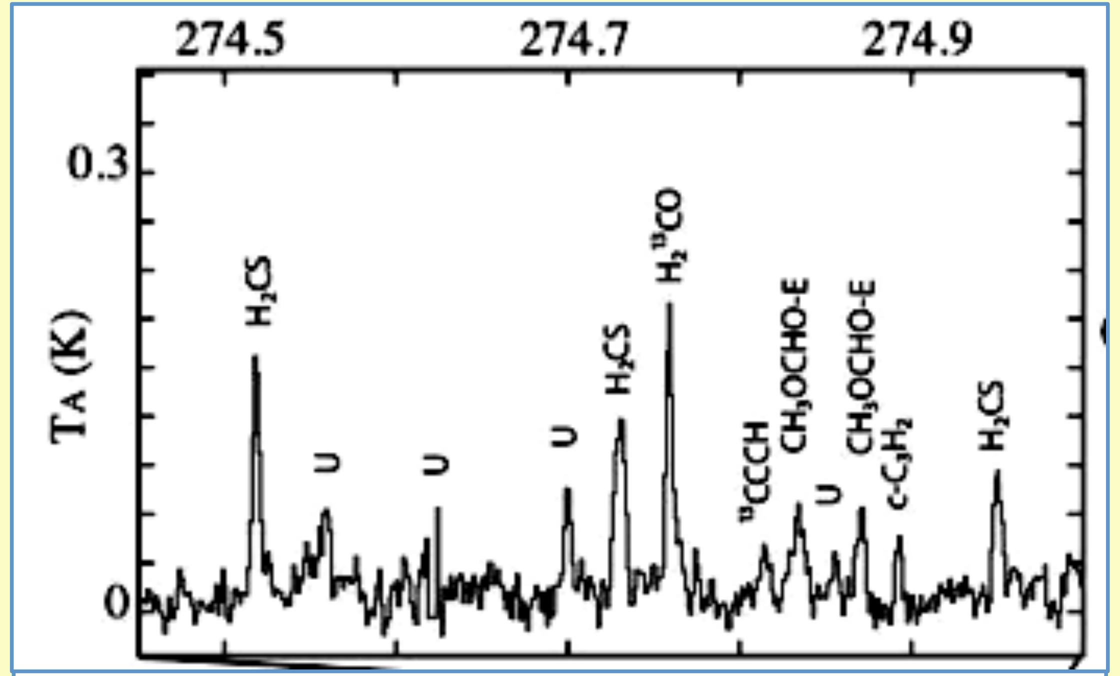
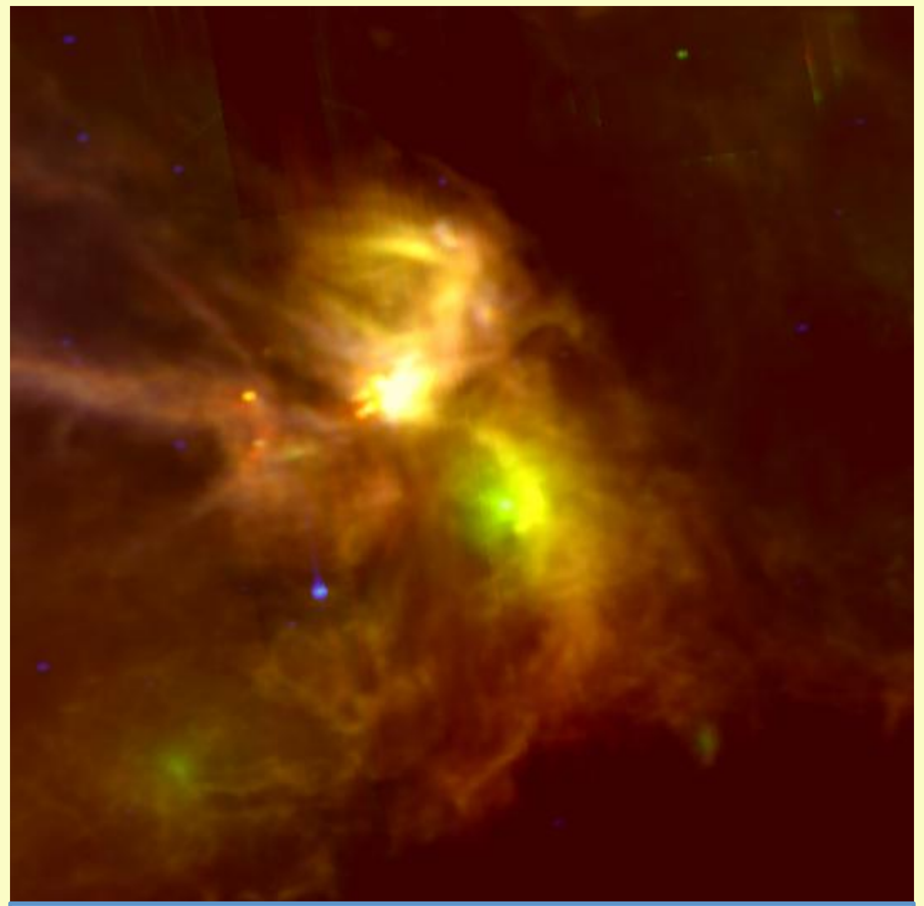
LINE IDENTIFICATION!



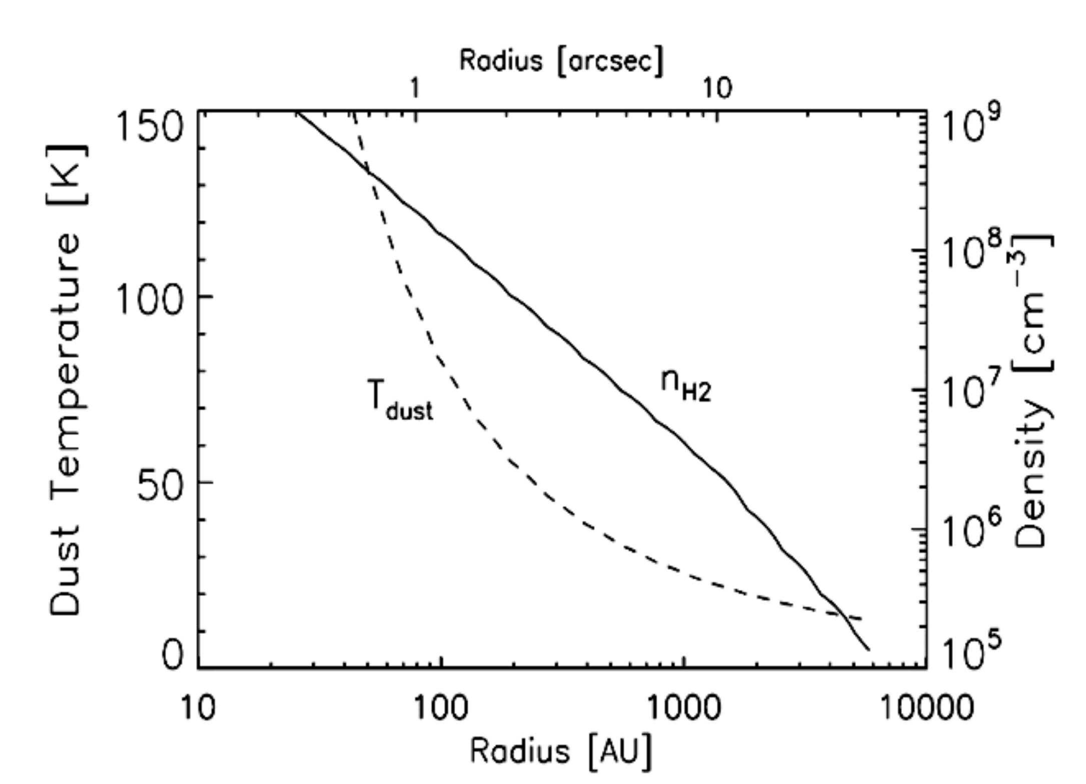
STEP 1: Observe the spectrum of the source.
Tool: telescope



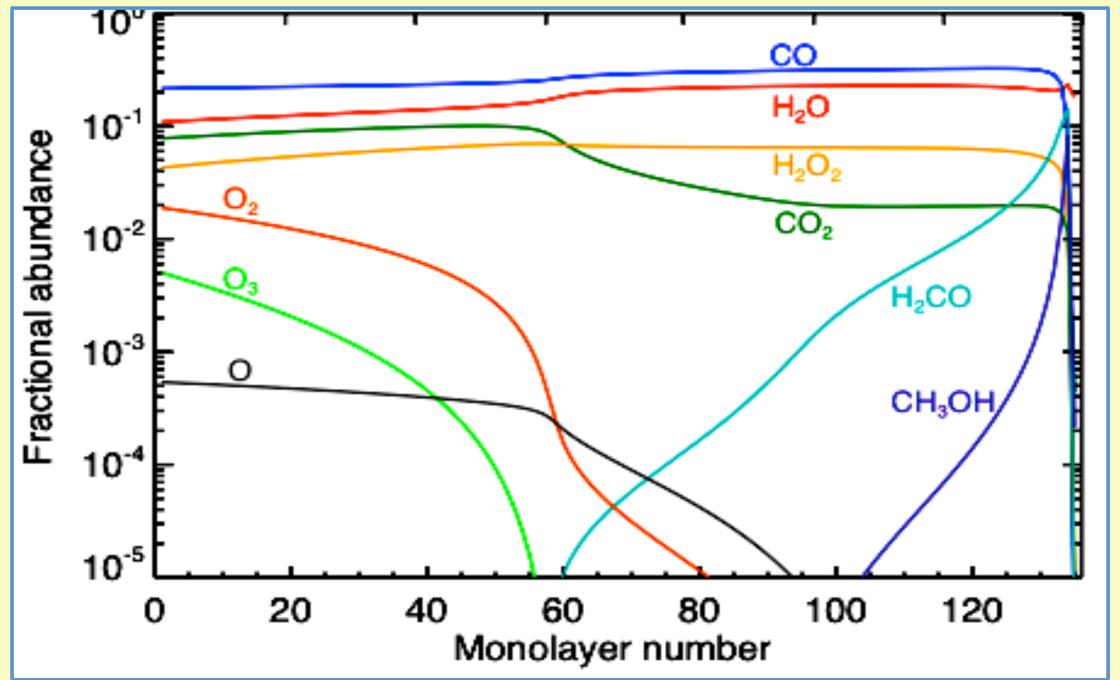
STEP 2: Identify the lines and species.
Tool: spectroscopic data



ASTROPHYSICAL OBJECT



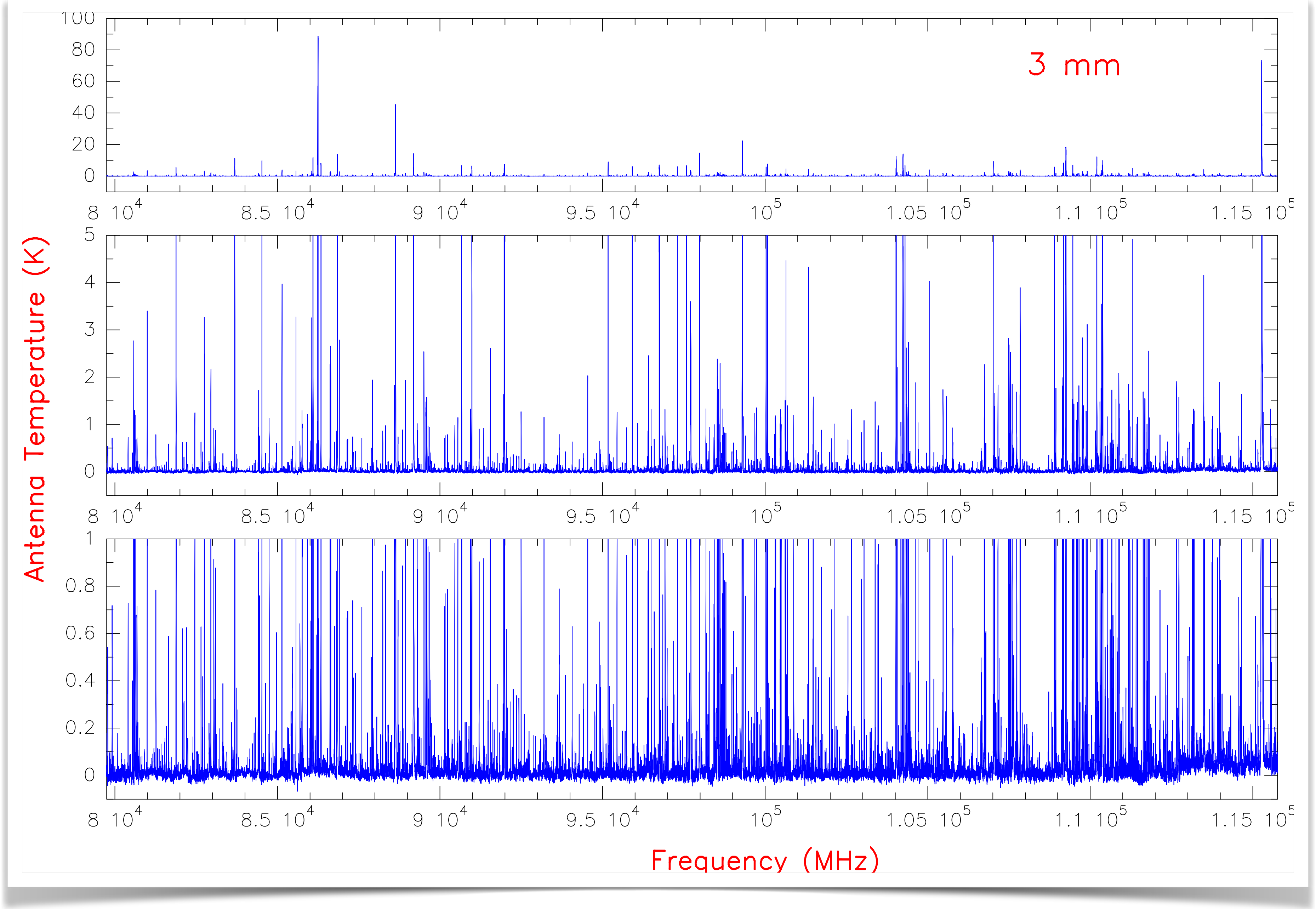
STEP 3: Derive the physical and chemical structure.
Tool: collisional coefficients



STEP 4: Understand the chemical structure.
Tool: reaction pathways and rate coefficients

Caselli & Ceccarelli 2012

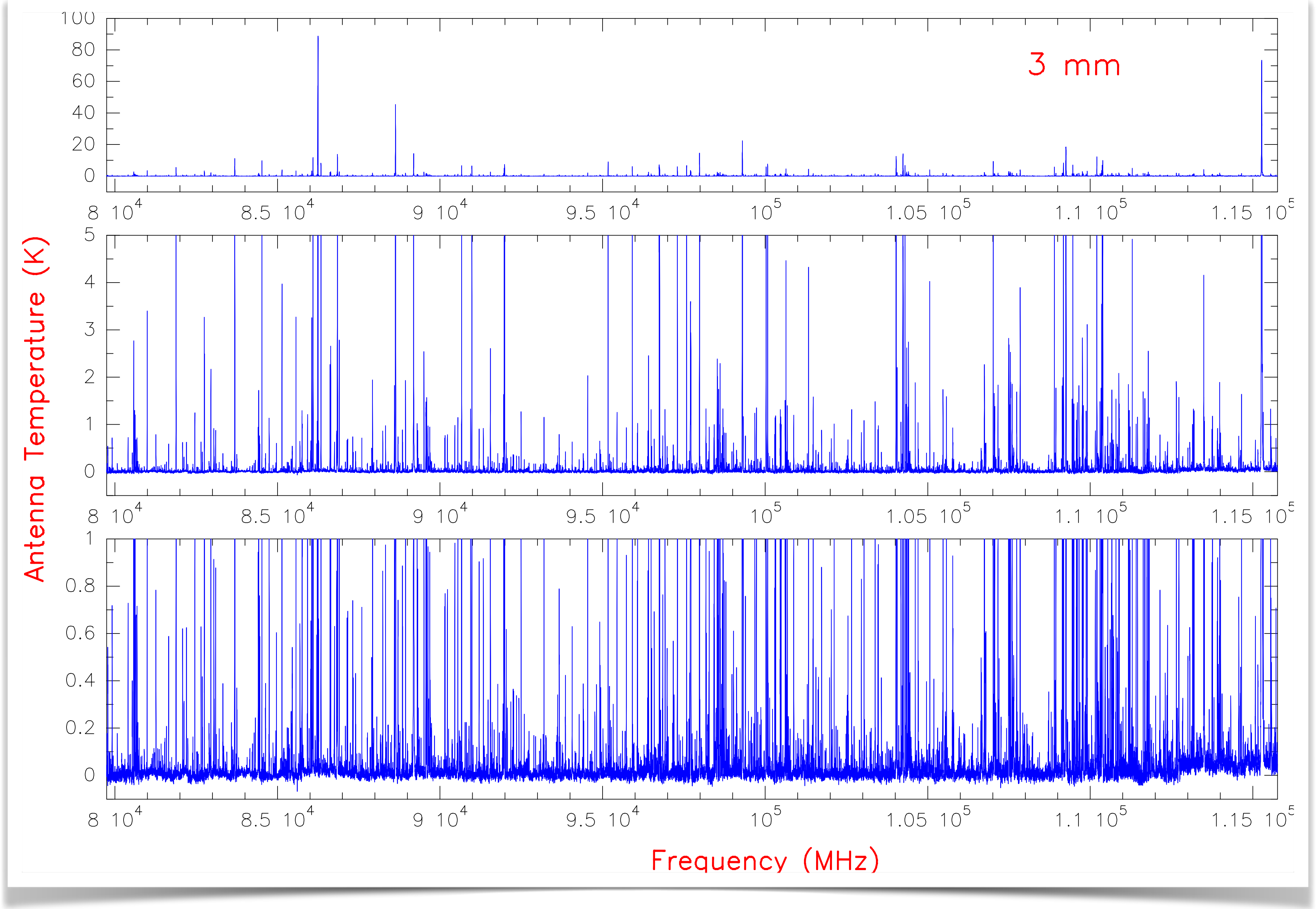
LINES, LINES, AND MORE LINES



Orion KL with IRAM 30m

Tercero+ 2010

LINES, LINES, AND MORE LINES



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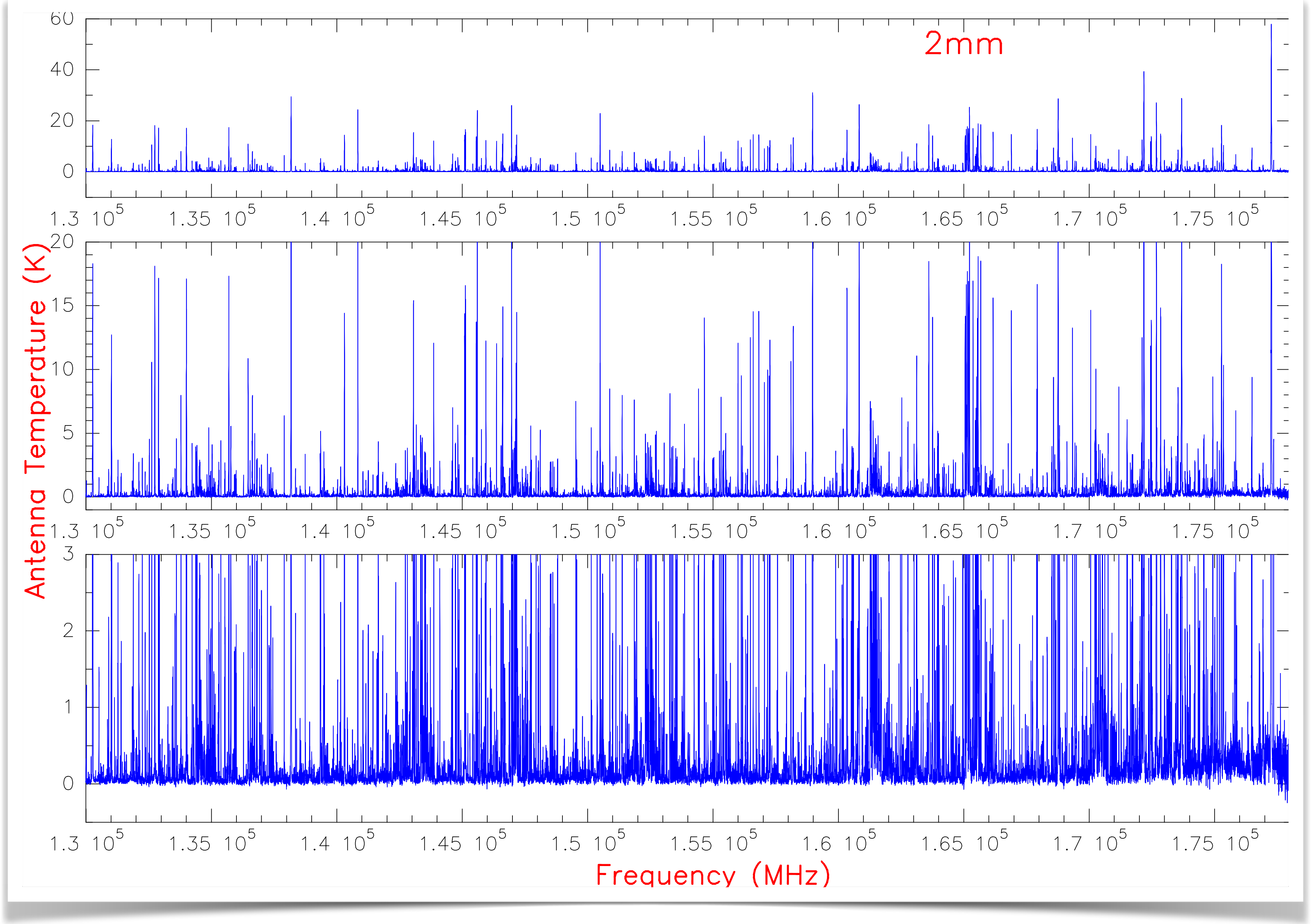

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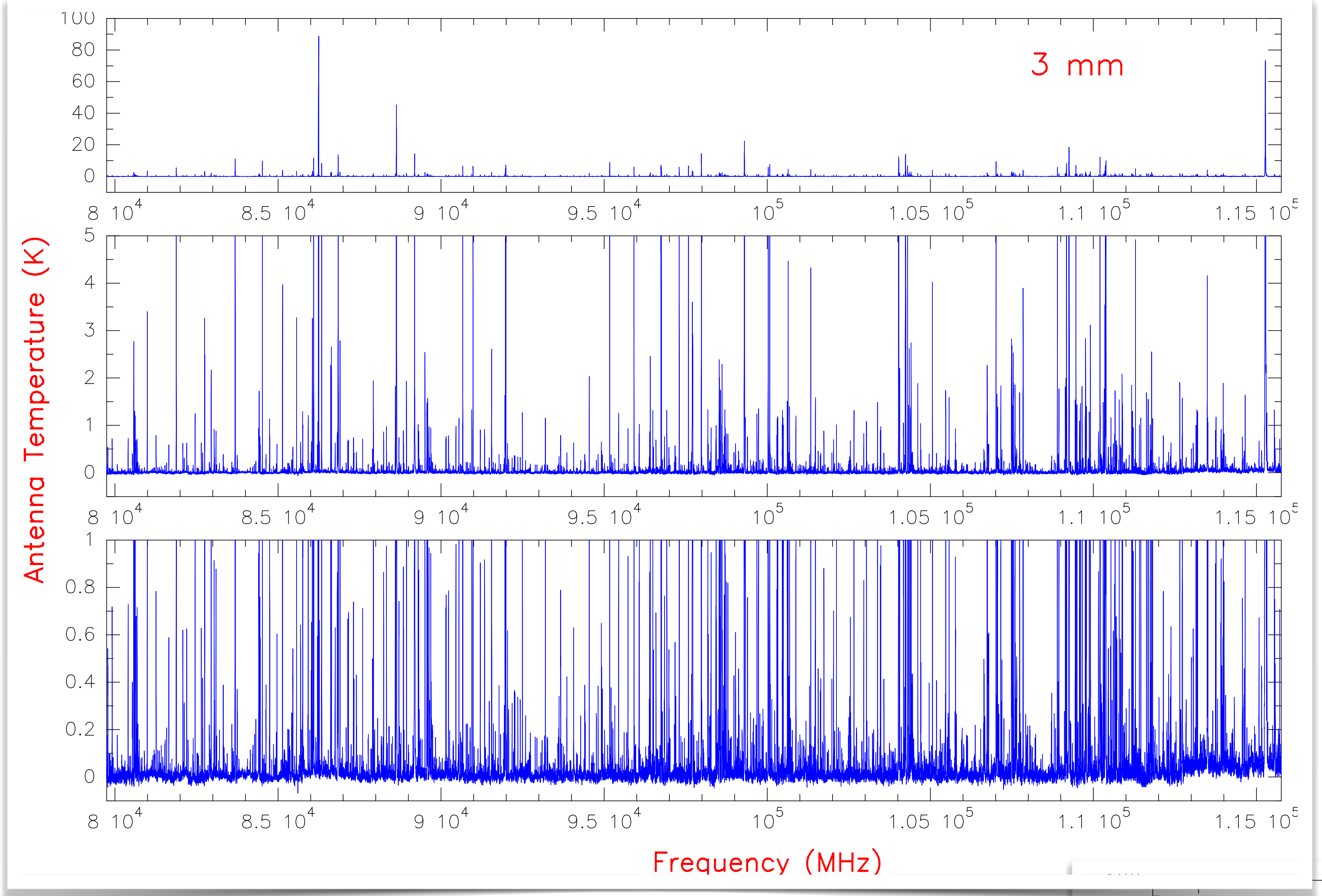

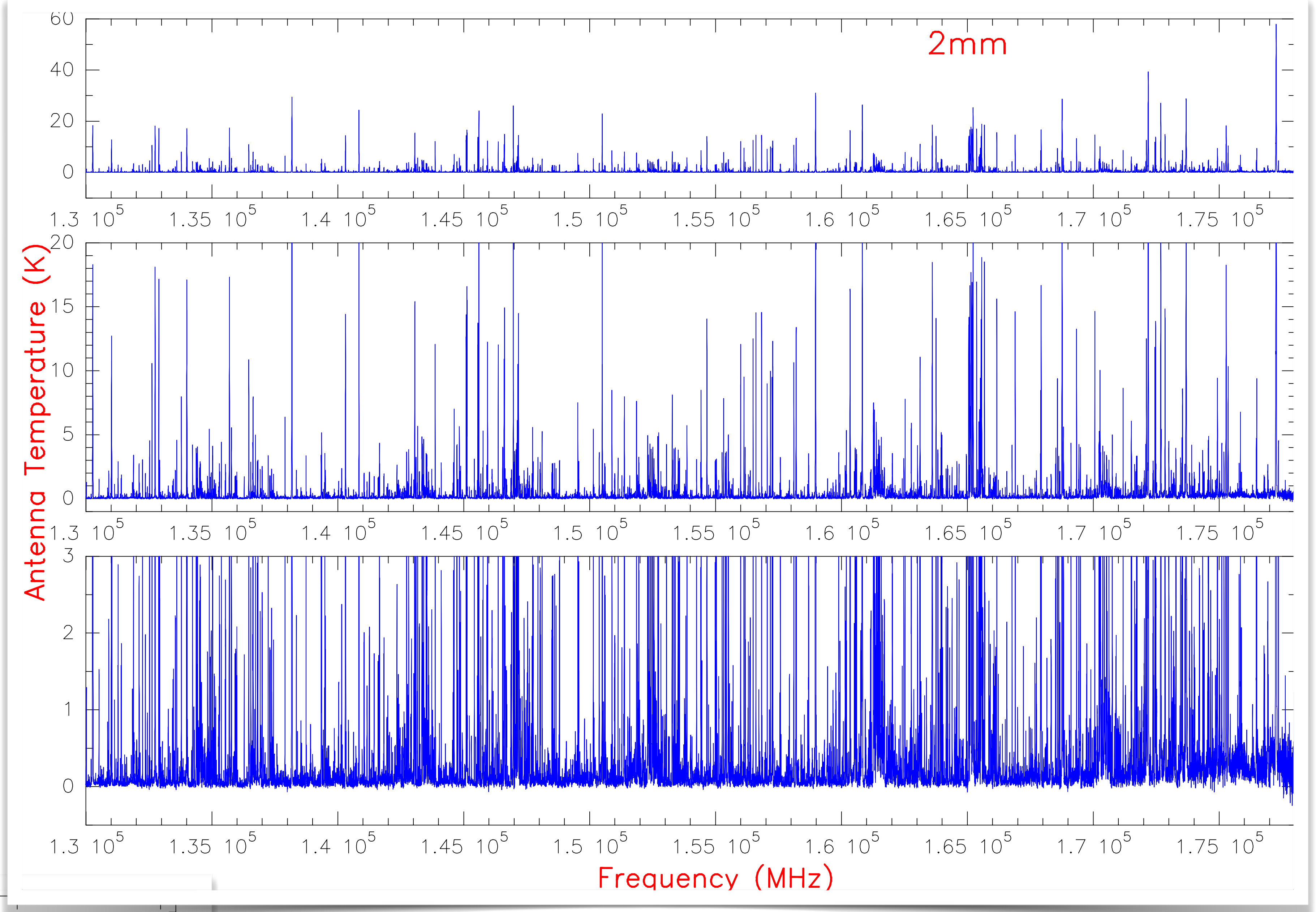

Orion KL with IRAM 30m

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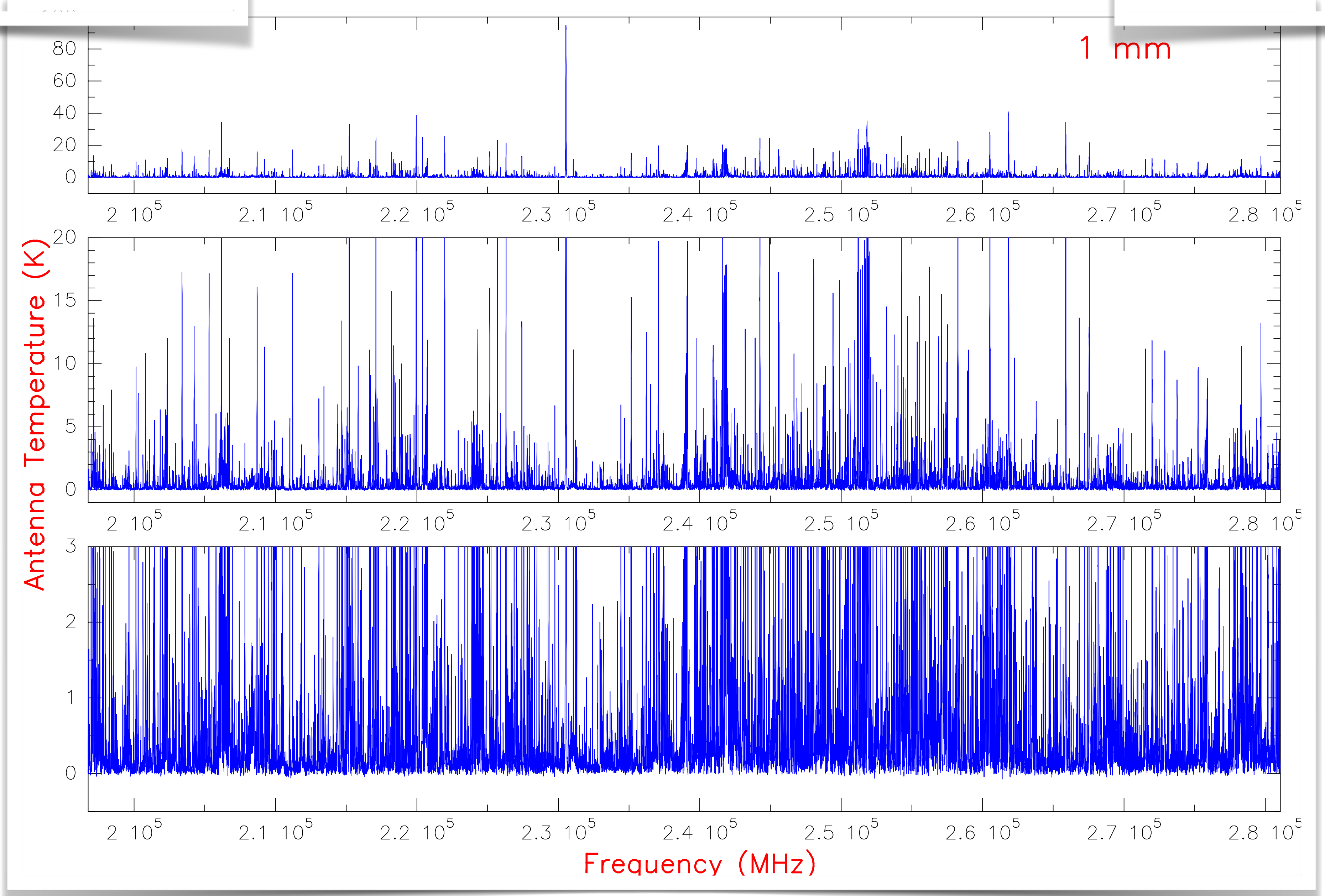
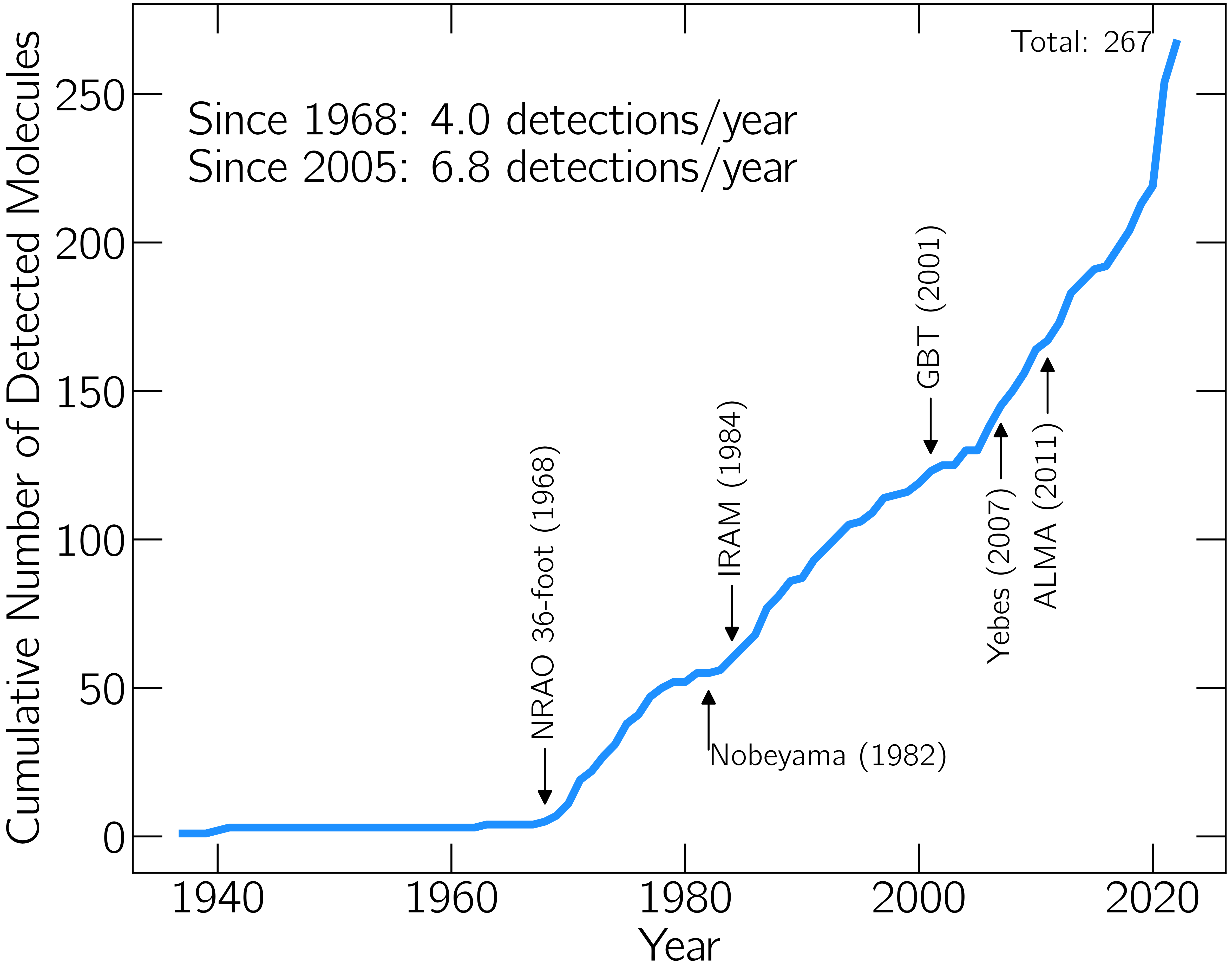


CUMULATIVE DETECTIONS



Two main surveys recently (both focused on TMC-1):

QUIJOTE (PI: J. Cernicharo) with Yebes 40m / GOTHAM (PI: B. McGuire) with GBT 100m

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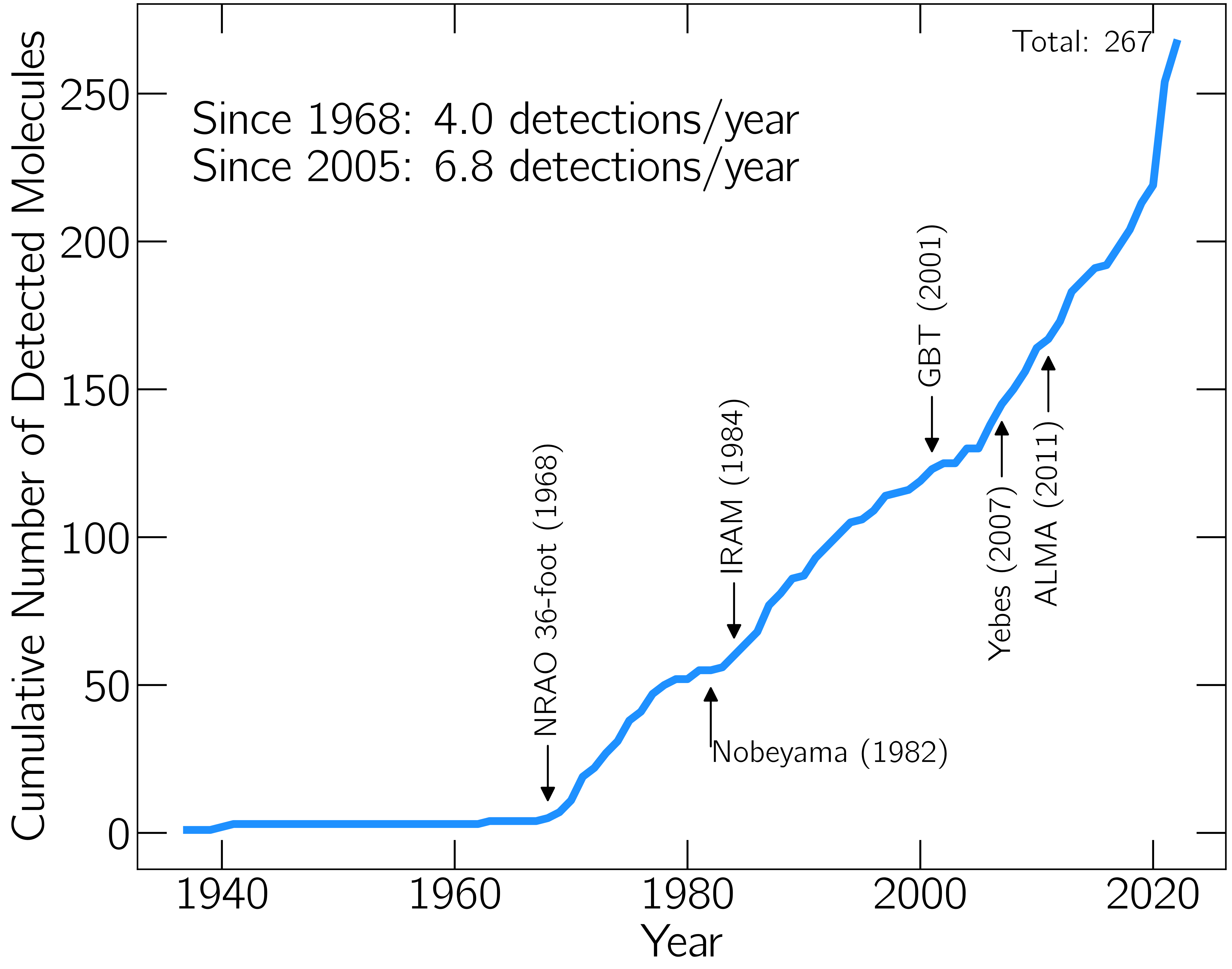

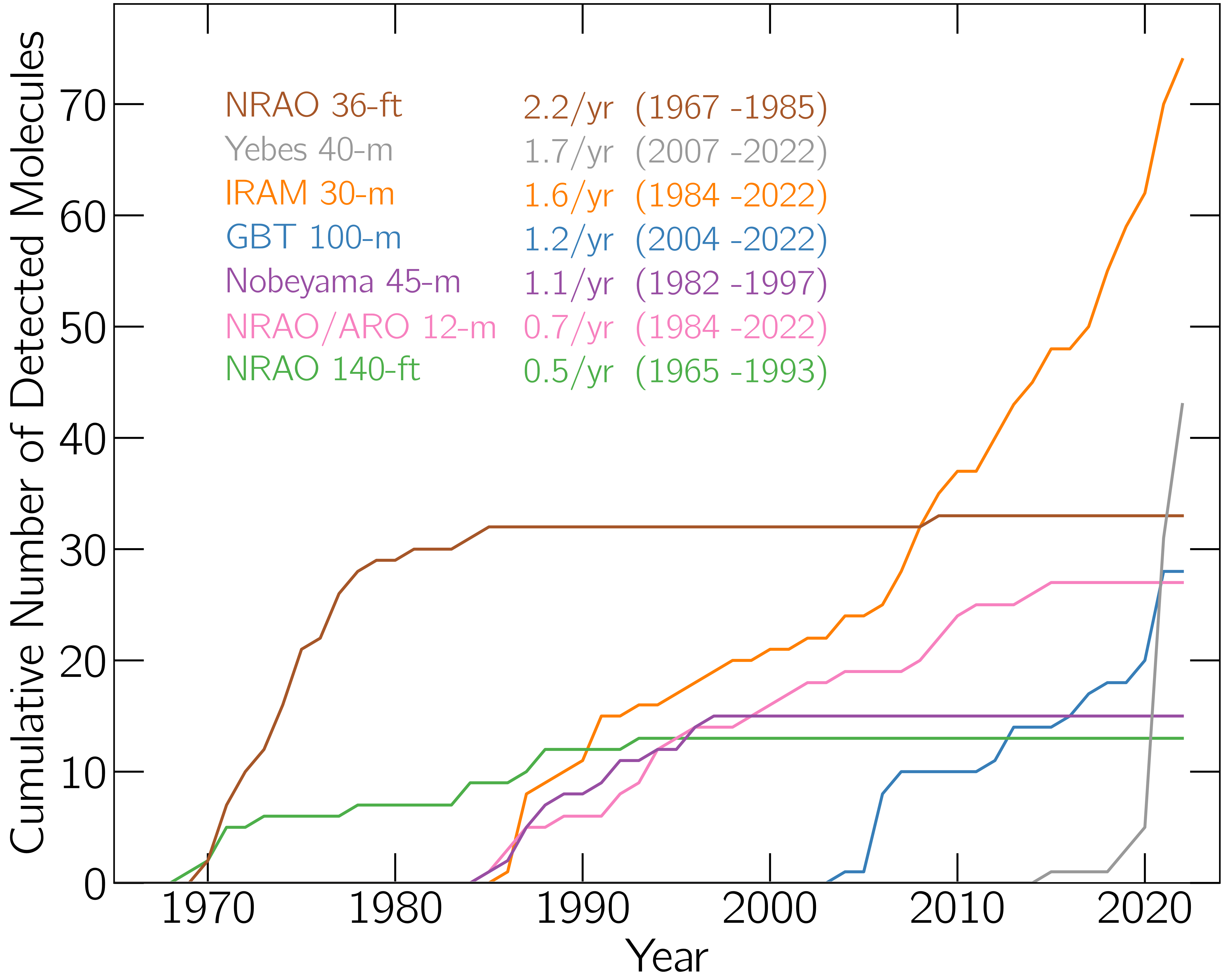

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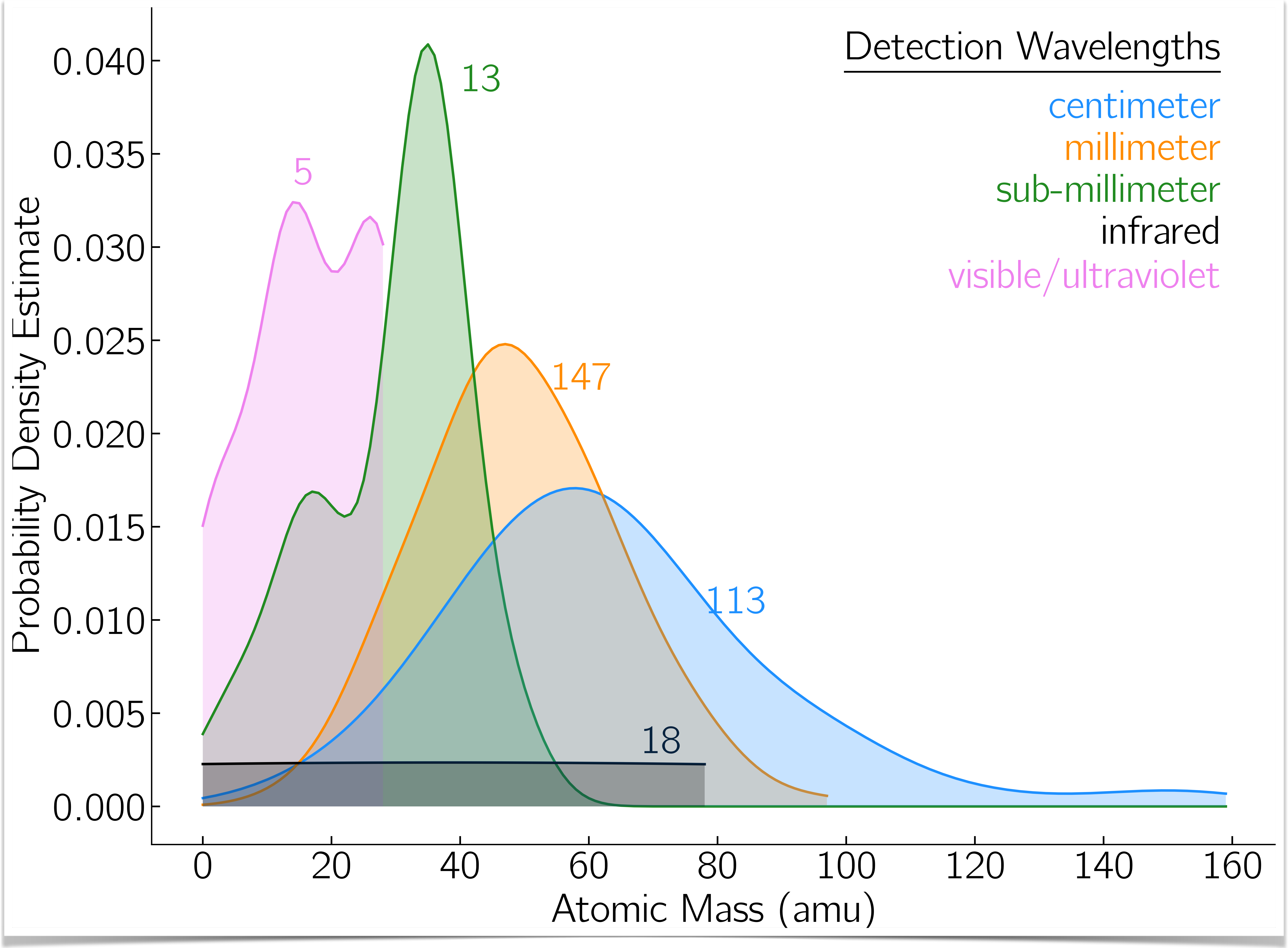
QUIJOTE (PI: J. Cernicharo) with Yebes 40m / GOTHAM (PI: B. McGuire) with GBT 100m

DETECTIONS AND WAVELENGTHS

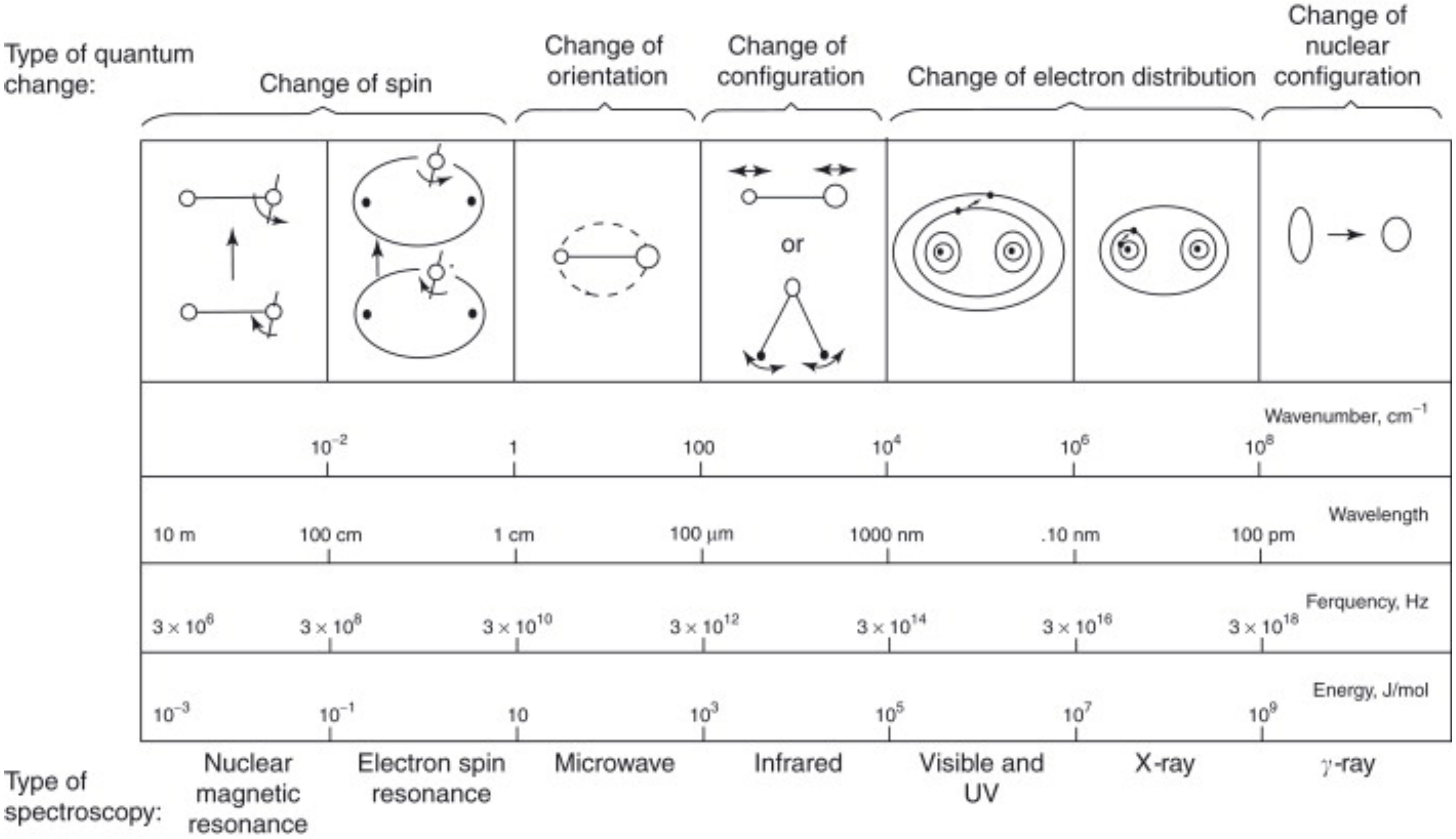


SPECTROSCOPY — BACK TO BASICS:

- “Spectroscopy is the general field of study that measures and interprets the electromagnetic spectra that result from the **interaction** between **electromagnetic radiation** and **matter** as a function of the wavelength or frequency of the radiation.” (*Wikipedia*)

- Depending on the frequency and matter we can have different “*kind*” of spectroscopy

ELECTROMAGNETIC SPECTRUM



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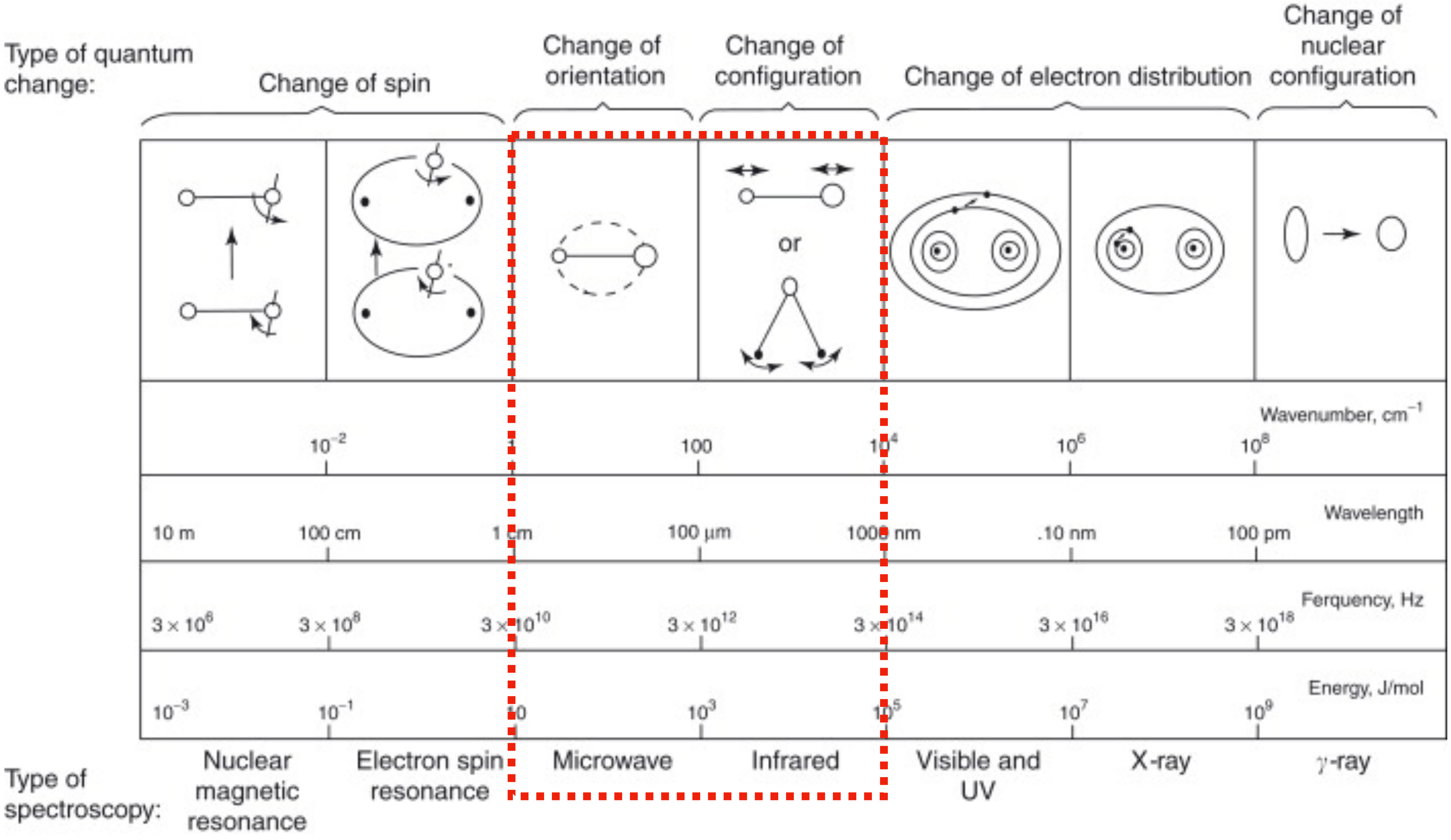
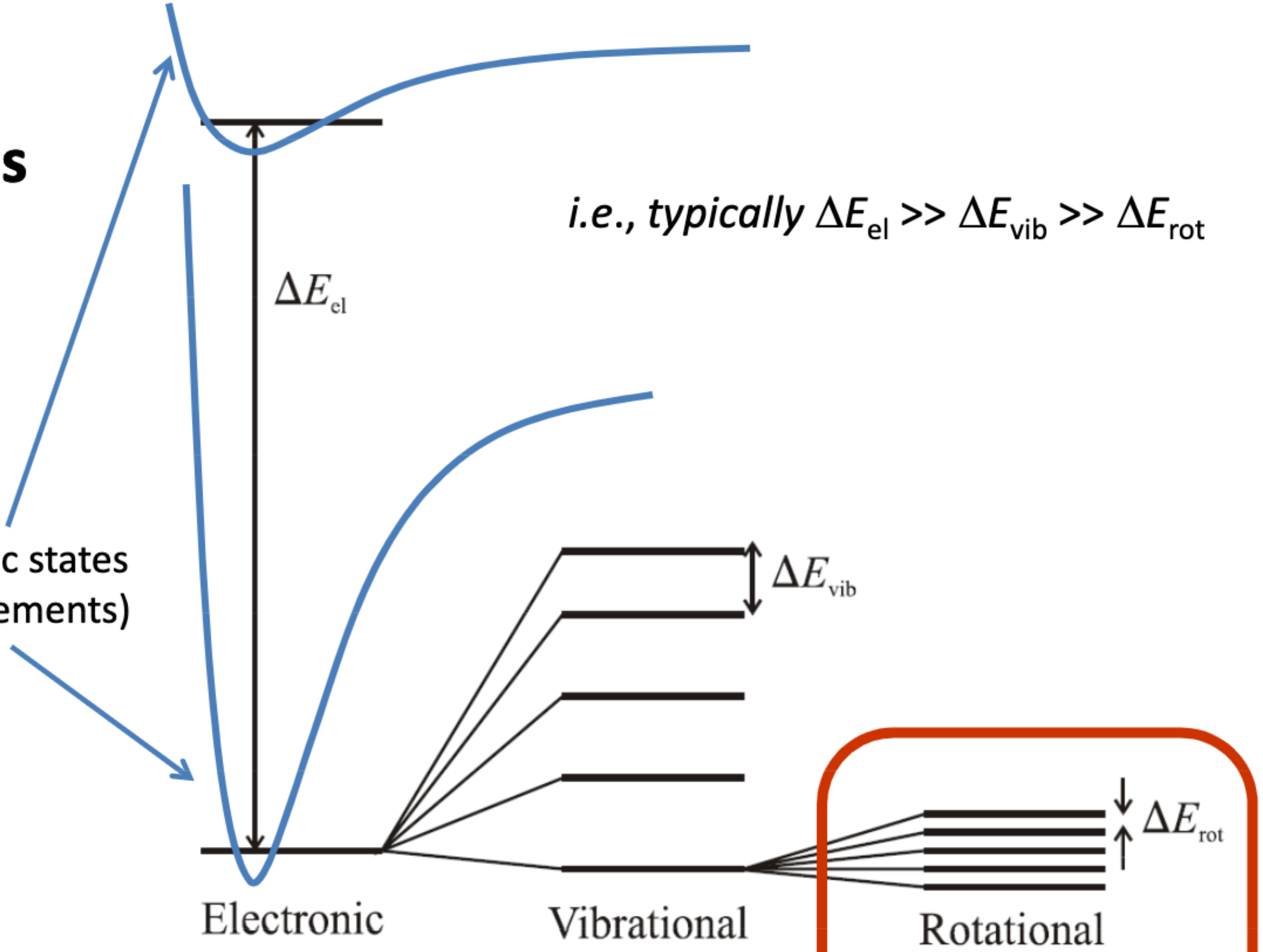


Molecular Energy Levels

i.e., typically $\Delta E_{el} >> \Delta E_{vib} >> \Delta E_{rot}$

Different electronic states
(electronic arrangements)



	Electronic	Vibrational	Rotational
$\Delta E \approx$	2 x 10^4 – 10^5 cm^{-1}	10^2 – 5 x 10^3 cm^{-1}	3 – 300 GHz (0.1 – 10 cm^{-1})
Transitions at $\lambda \approx$	500 – 100 nm	100 μm – 2 μm	10 cm – 1 mm
	Vis – UV	infrared	microwave

(SUB)MILLIMETER VS INFRARED

- **(Sub)millimeter**:

  A. Very high spectral resolution ($R > 10^6$)

  B. Probing gas-phase w/ very low abundance (down to 10^{-12} w.r.t. H_2)

  C. Mapping regions

- **Infrared**:

  A. Moderate spectral resolution ($R \sim 10^3 - 10^4$)

  B. Gases and solids w/ abundances (down to 10^{-8} w.r.t. H_2)

  C. Molecules w/o permanent dipole moment ($C_2H_2, CH_4, CO_2, \ldots$)

IR REVOLUTION WITH JWST IS HAPPENING!

- Things are going to change very soon thanks to Webb.
- Unprecedented sensitivity.
- => very good for lab!!! => new experiments needed?
- Very likely will revolutionise the gas-grain chemistry as we know it so far.

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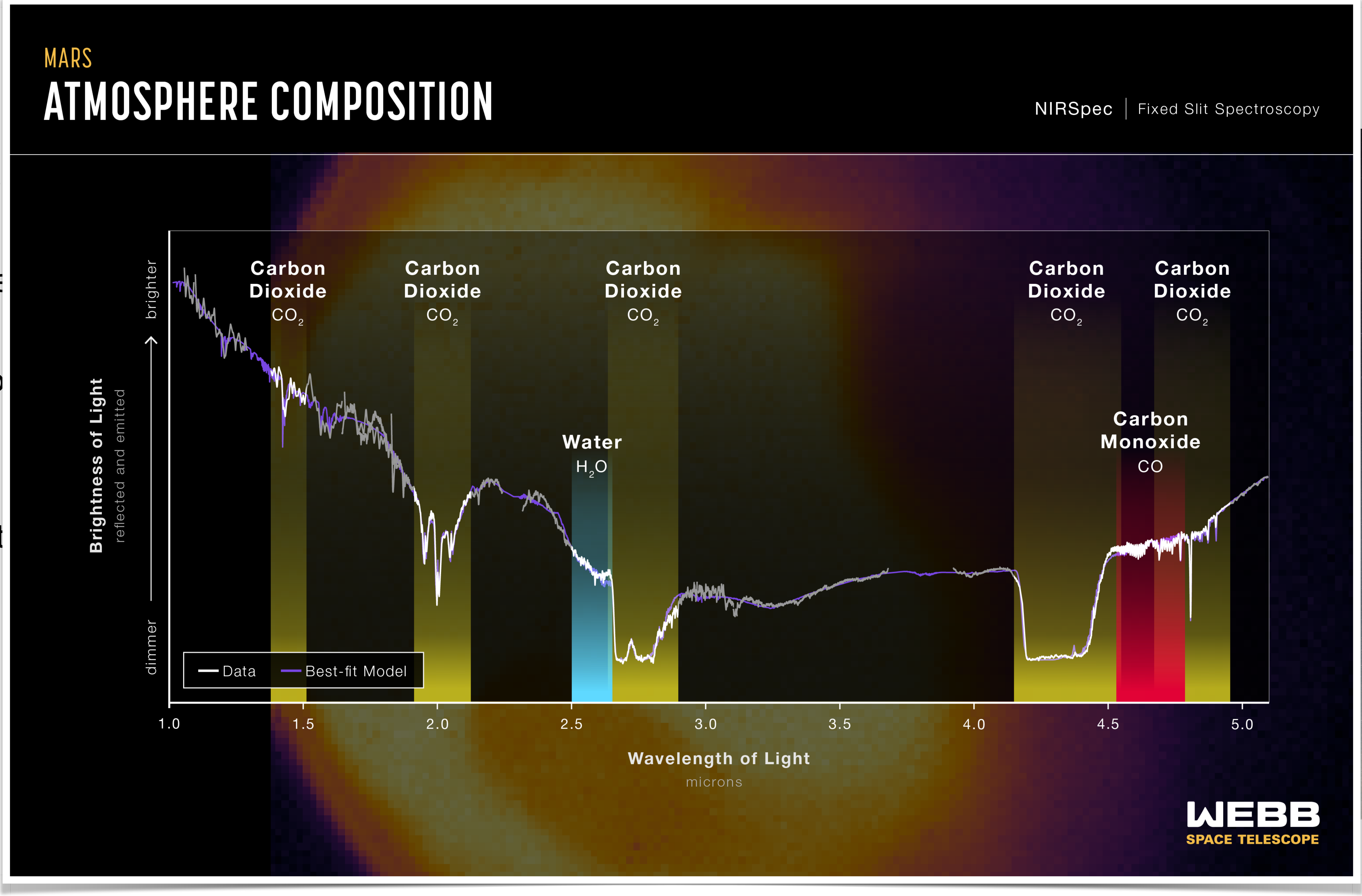
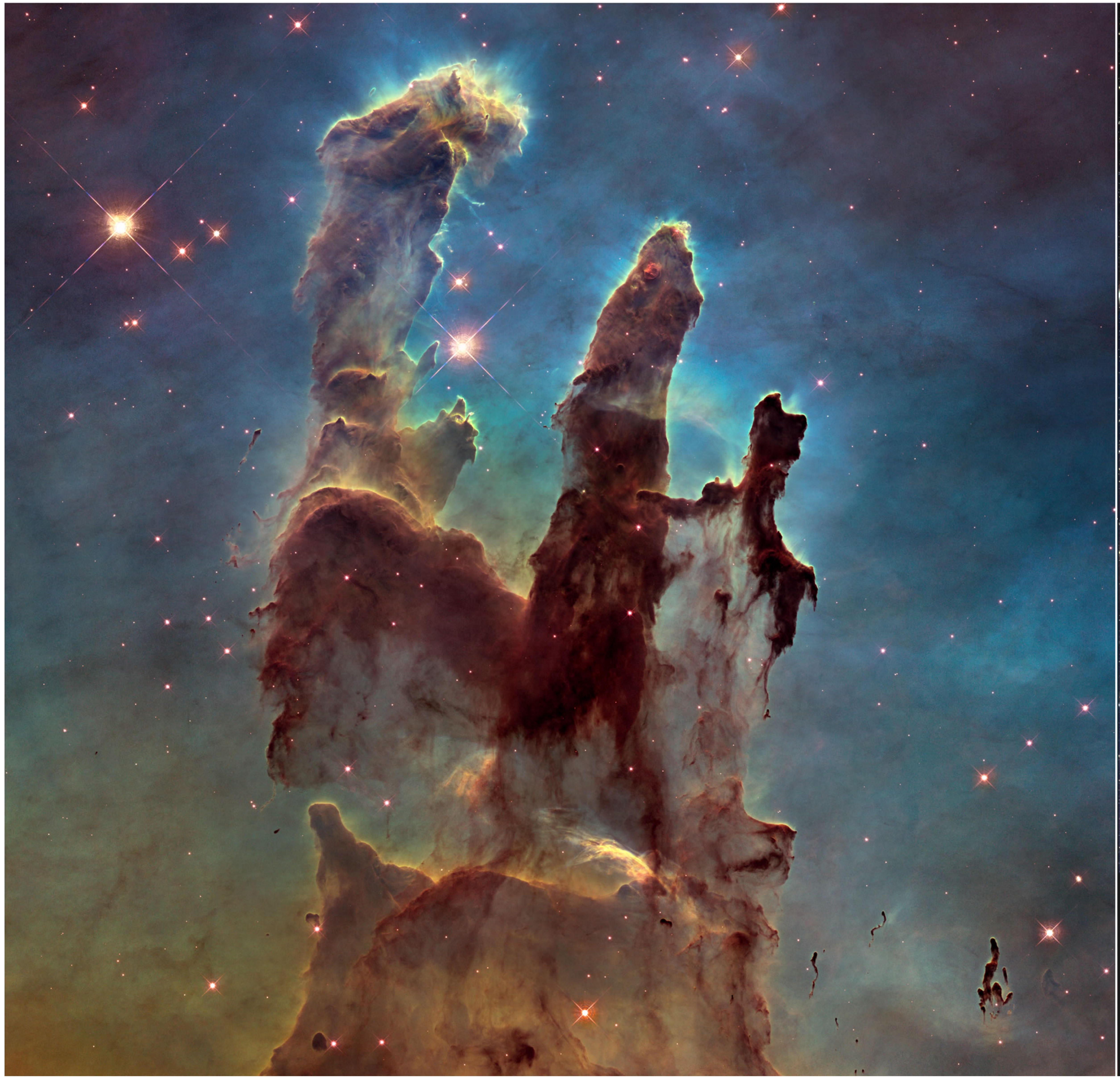


Hubble



JWST

- The protostar within the dark cloud L1527 is embedded within a cloud of material feeding its growth. Ejections from the star have cleared out cavities above and below it, whose boundaries glow orange and blue in this infrared view. The upper central region displays bubble-like shapes due to stellar “burps,” or sporadic ejections. Webb also detects filaments made of molecular hydrogen that has been shocked by past stellar ejections. The edges of the cavities at upper left and lower right appear straight, while the boundaries at upper right and lower left are curved. The region at lower right appears blue, as there’s less dust between it and Webb than the orange regions above it.

- Release date: 16/11/2022

SO, WE NEED LINE CATALOGUES AND WE NEED HIGH RESOLUTION!

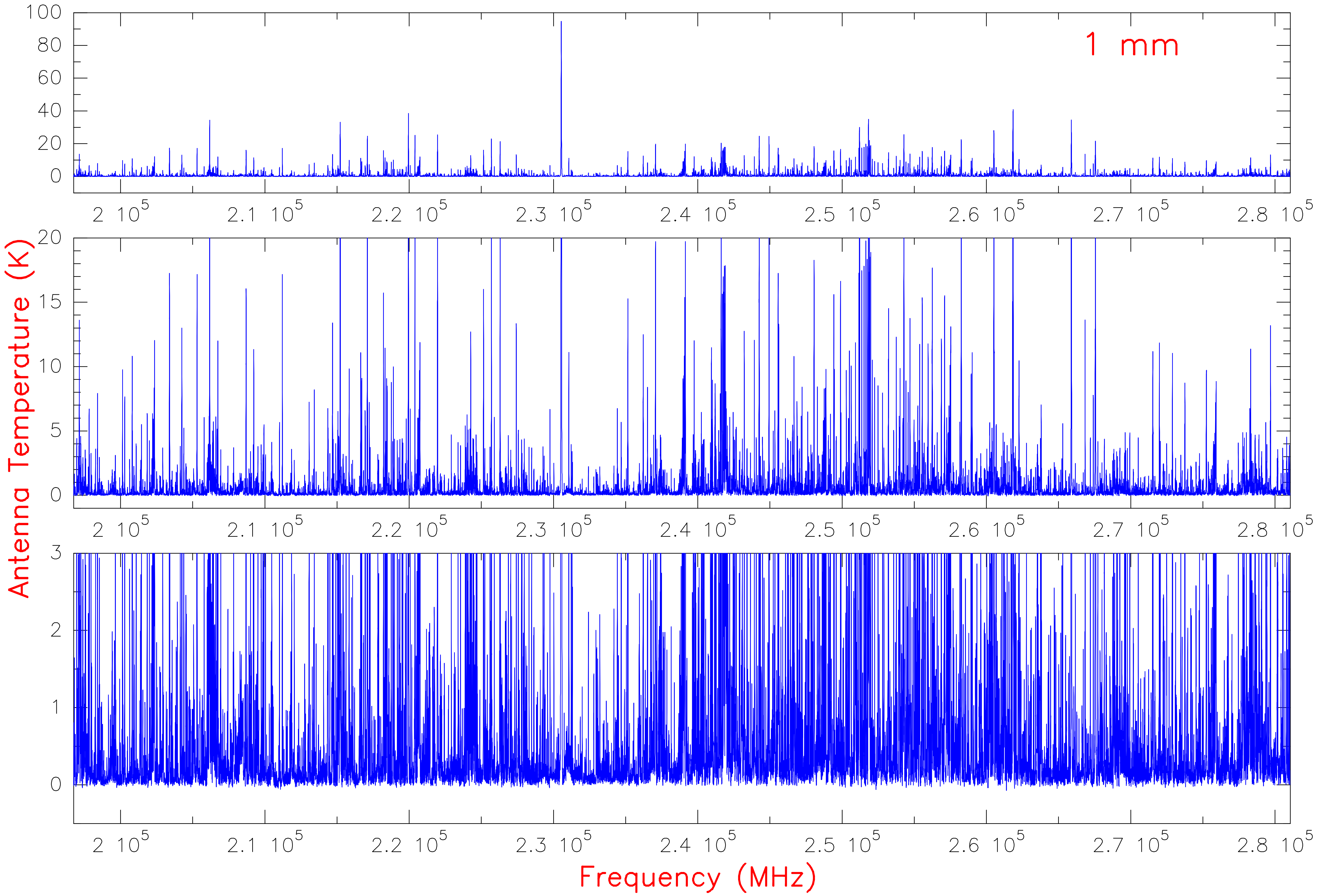
- Modern Radioastronomical facilities have

  A. **<u>amazing sensitivity</u>** => new species, isotopologues, vibrational excited states

  B. **<u>unprecedented bandwidth</u>** => more transitions, unbiased survey, more molecules

  C. **<u>higher velocity/frequency resolution</u>** => high resolution catalogues needed

- Confusion limited!



ROTATIONAL SPECTROSCOPY

Microwave Molecular Spectra — W. Gordy and R.L. Cook (John Whiley and Sons, Inc, 1984)

Spectra of Atoms and Molecules — P.F. Bernath (Oxford University Press, 2005)

Molecular Rotation Spectra — H.W. Kroto (Dover Publications Inc. 1992)

Symmetry and Spectroscopy — D.C. Harris and M.D. Bertolucci (Dover Publications Inc. 1989)

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- Replacing classic observables with corresponding operator:

$$P_x \rightarrow \hat{J}_x = \sum_n \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right); \quad P_y \rightarrow \hat{J}_y = \sum_n \frac{\hbar}{i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right); \quad P_z \rightarrow \hat{J}_z = \sum_n \frac{\hbar}{i}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

RIGID ROTOR APPROXIMATION - II

- The resulting Hamiltonian operator is: $\hat{H} = \frac{\hat{J}_a^2}{2I_a} + \frac{\hat{J}_b^2}{2I_b} + \frac{\hat{J}_c^2}{2I_c}$
- The three primary axes defined in body-fixed coordinates *a,b,c,* and their corresponding moments of inertia I_i are found by diagonalising the 3D moment of inertia tensor.
- The **<u>rotational constants</u>** *A, B,* and *C* are defined in terms of these I_i values as:

$$A = \frac{\hbar^2}{2I_a}, \quad B = \frac{\hbar^2}{2I_b}, \quad C = \frac{\hbar^2}{2I_c}$$

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RIGID ROTOR APPROXIMATION - III

- Depending on the symmetry and structure of the molecule, different rotor (or "top"):

  1. Spherical Top: $I_a = I_b = I_c$ (e.g. CH_4)
  2. Linear Rotor: $I_a = 0;\ I_b = I_c$ (e.g. HCN)
  3. Symmetric Oblate Top: $I_a = I_b < I_c$ (e.g. NH_3)
  4. Symmetric Prolate Top: $I_a < I_b = I_c$ (e.g. CH_3CN)
  5. Asymmetric Top: $I_a \neq I_b \neq I_c$ (e.g. CH_3OH)

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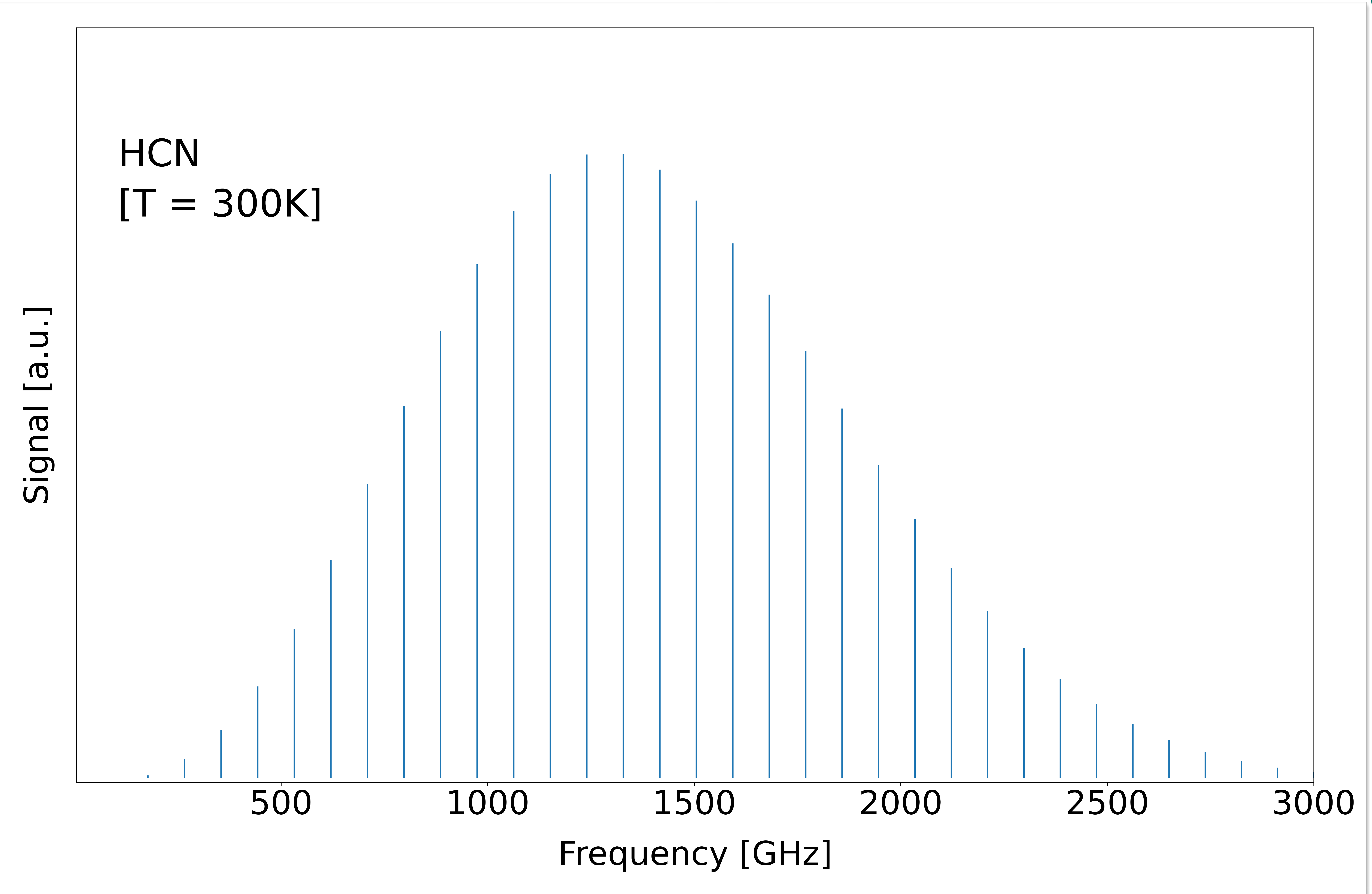
- Rotational spectrum consists of a harmonic series of lines having frequencies $\nu = 2B, 4B, 6B, \ldots$

HYDROGEN CYANIDE



LINE INTENSITIES

- The line intensity is determined by the product of the transition probability P_{mn} and the population difference Δp between a lower and an upper energy state, where P_{mn} is:

$$P_{mn} = |\langle \psi_m | \hat{\mu} | \psi_n \rangle|^2$$

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"Larger" the molecule —> Smaller rotational constants —> Denser the spectrum!

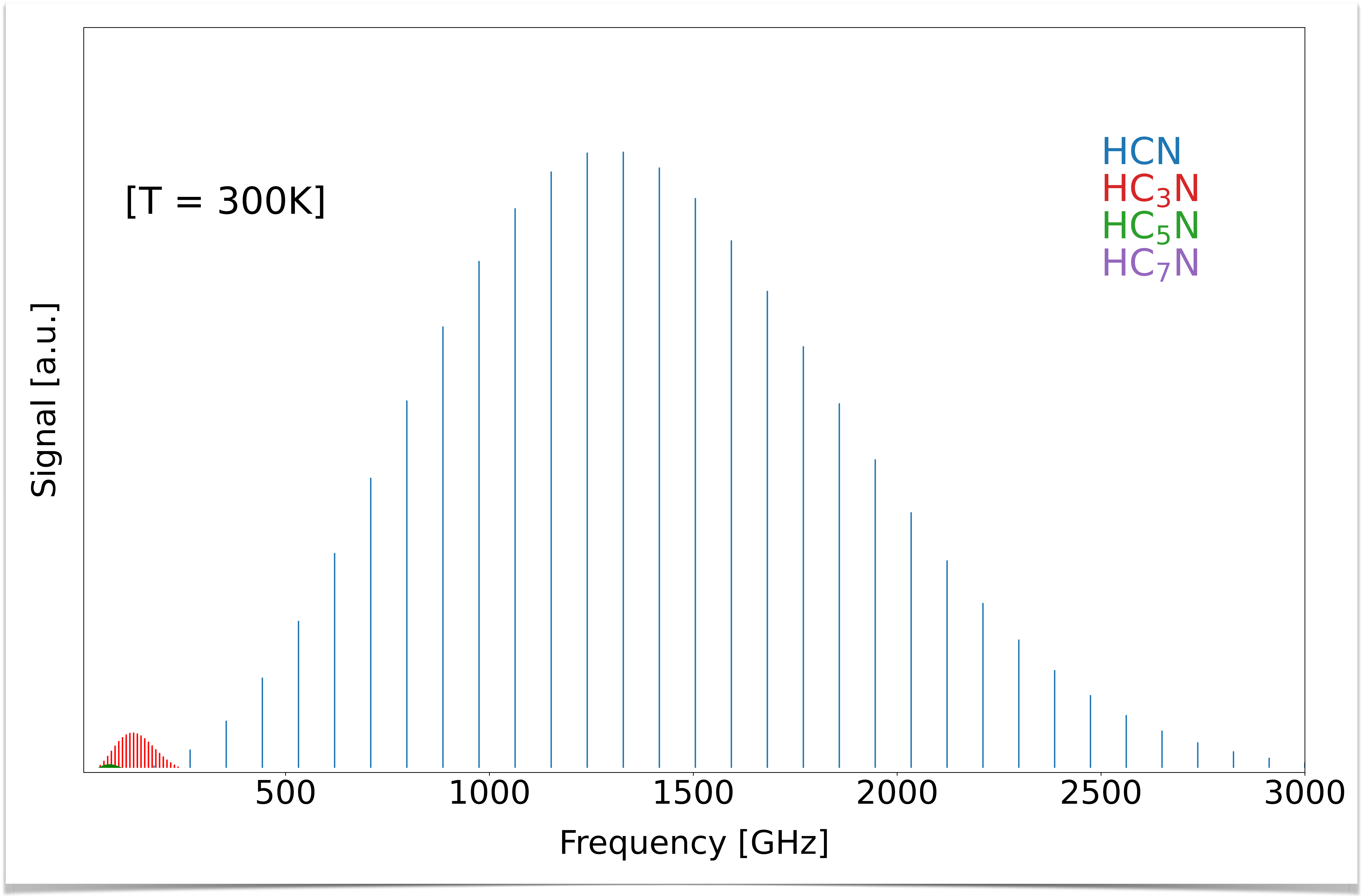
LINEAR ROTORS MIGHT BE NOT SO SIMPLE AFTER ALL...

Rotational Constants and Number of Energy Levels below kT at 150 K for All HC_xN Species Detected in the ISM

Molecule	B (MHz)	# Levels[a] $< kT$ @ 150 K	Lab Ref.
HCN	44316	18	1
HC_2N	10986	36	2
HC_3N	4549	58	3
HC_4N	2302	80	4
HC_5N	1331	107	5
HC_7N	564	166	6
HC_9N	290	231	7

McGuire 2022

CYANOPOLYYNES



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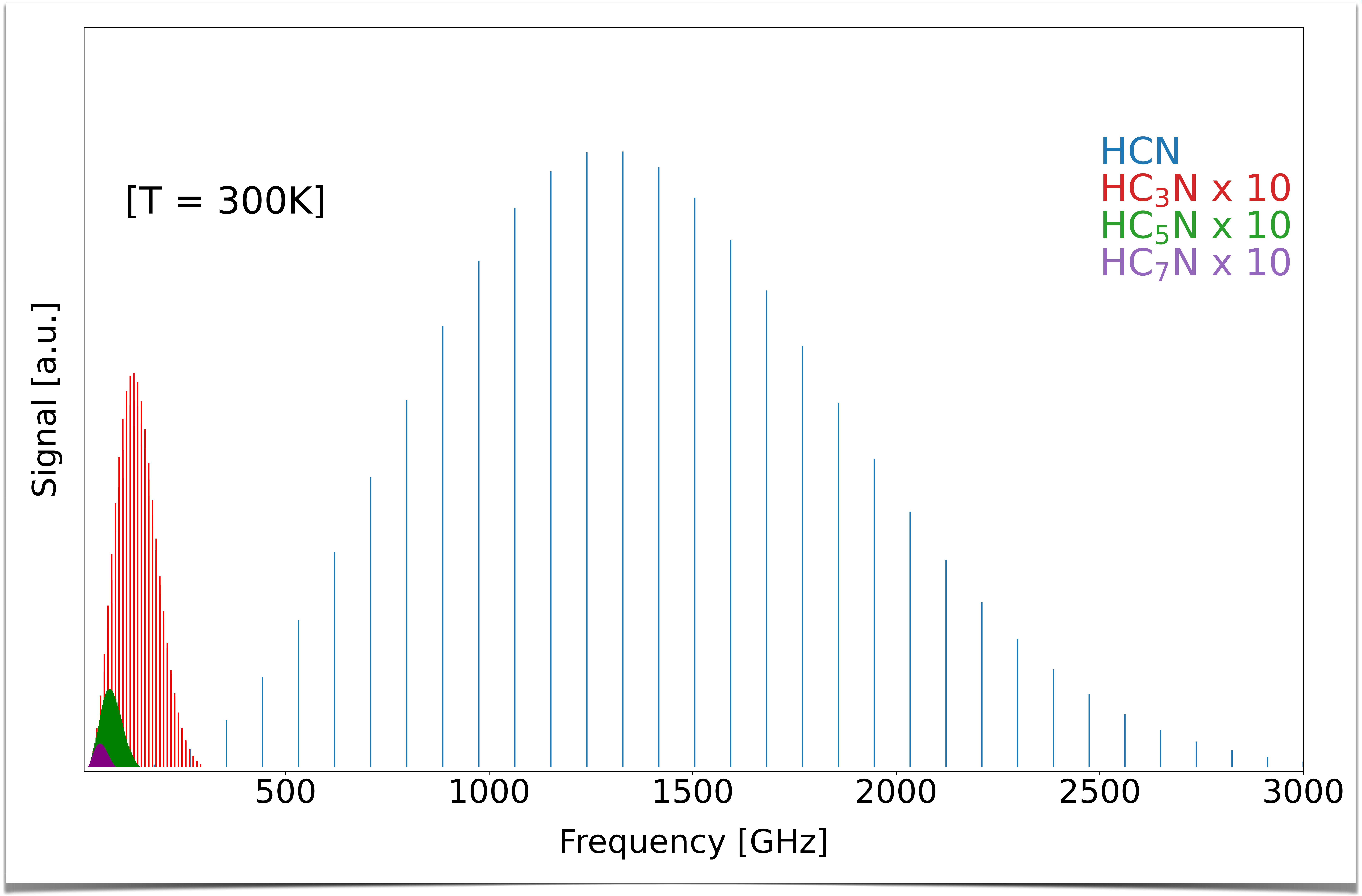

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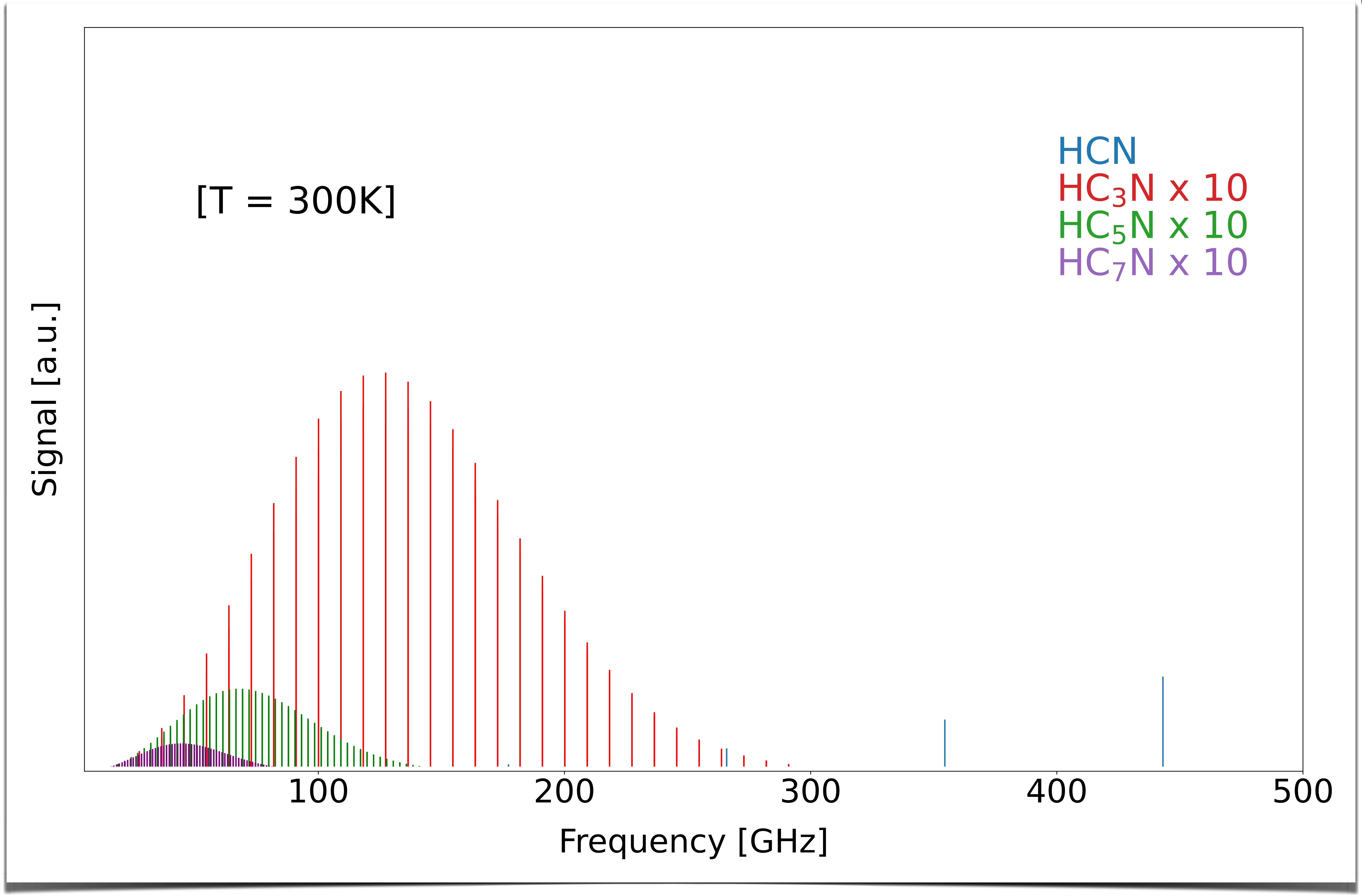

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- Selection rules are now $\Delta J = \pm 1$ and $\Delta K_c = 0$

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- Hence: $\hat{H} = \frac{\hat{J}_a^2}{2I_a} + \frac{\hat{J}_b^2}{2I_b} + \frac{\hat{J}_c^2}{2I_c} = (\frac{1}{2I_a} - \frac{1}{2I_b})\hat{J}_a^2 + \frac{\hat{J}^2}{2I_b}$ (where $\hat{J}^2 = \hat{J}_a^2 + \hat{J}_b^2 + \hat{J}_c^2$)
- Solving the Schrödinger's equation we have:

$$E^{pr}(J, K_a) = \frac{\hbar^2 J(J+1)}{2I_b} + (\frac{\hbar^2}{2I_a} + \frac{\hbar^2}{2I_b})K_a^2 \equiv BJ(J+1) + (A-B)K_a^2$$

- K_a describes the projection of the angular momentum J on the symmetry axis c $-->$ K-degeneracy!
- Rotational levels labelled J_{K_a} with $J = 0, 1, 2,...$ and $K_a = 0 \pm 1, \pm 2, \ldots, \pm J$
- Selection rules are now $\Delta J = \pm 1$ and $\Delta K_a = 0$

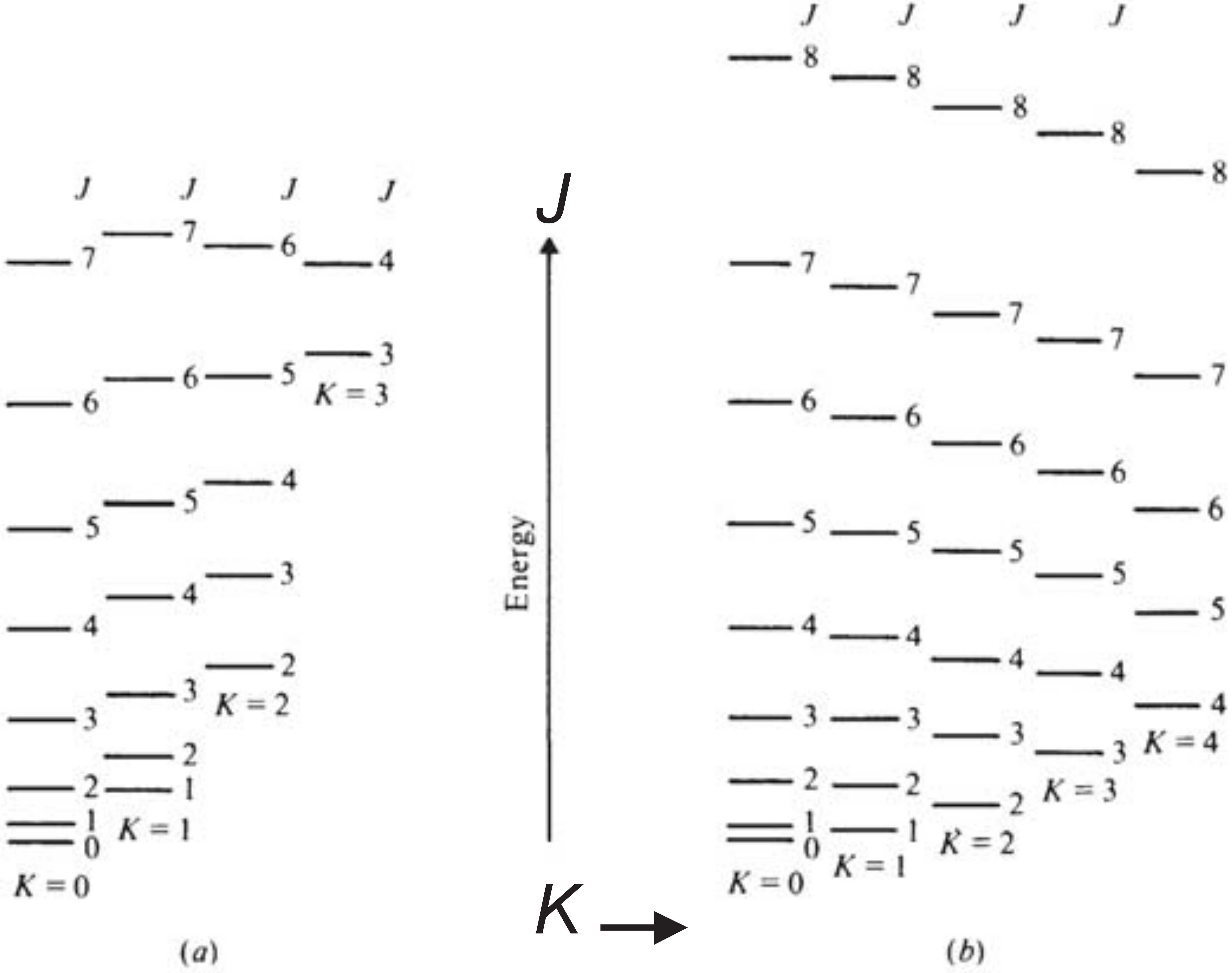
SYMMETRIC TOPS – THE PROLATE CASE

Prolate

$$E = BJ(J+1) + (A-B)K^2$$

Oblate

$$E = BJ(J+1) + (C-B)K^2$$



ASYMMETRIC TOPS

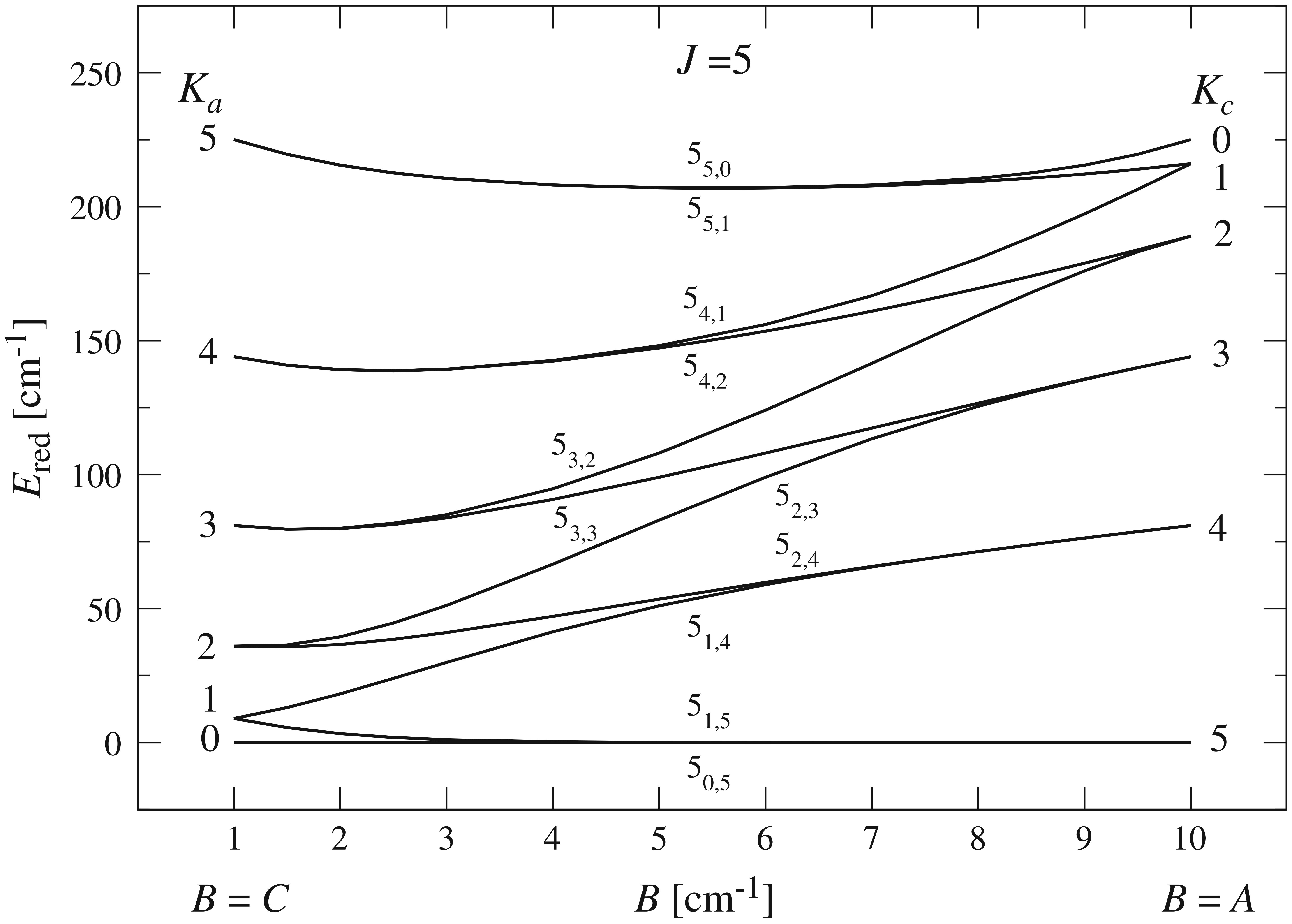
- Now $I_a \neq I_b \neq I_c$
- The Schrödinger equation has no general analytical solutions!
- It can be solved using a symmetric top basis set and changing the form of the terms on the Hamiltonian operator.
- The degree of asymmetry is quantified by Ray's asymmetry parameter $\kappa = \dfrac{2B - A - C}{A - C}$

which runs from -1 for a prolate top to +1 for an oblate top.

- Levels are now labelled J_{K_a,K_c}, where now only J is a "good" quantum number, while K_a and K_c are just for labels (they become good quantum numbers in the prolate and oblate symmetric top limit, respectively).

ASYMMETRIC TOPS

Scheme of the energy levels of an asymmetric top, plotted for J=5, with the constant B varying continuously from C (prolate limit) to A (oblate limit).



Yamada & Winnewisser, 2011

ASYMMETRIC TOPS

- In general three non-vanishing dipole moments components $\mu_a, \mu_b,$ and μ_c.
- Selection rules are $\Delta J = -1, 0, +1$ (*P*-branch, *Q*-branch, *R*-branch).

Transition	Dipole moment component	ΔK_a [a]	ΔK_c [a]
a-type	$\mu_a \neq 0$	$0, (\pm 2, \pm 4, \ldots)$	$\pm 1, (\pm 3, \pm 5, \ldots)$
b-type	$\mu_b \neq 0$	$\pm 1, (\pm 3, \pm 5, \ldots)$	$\pm 1, (\pm 3, \pm 5, \ldots)$
c-type	$\mu_c \neq 0$	$\pm 1, (\pm 3, \pm 5, \ldots)$	$0, (\pm 2, \pm 4, \ldots)$

[a] The transitions in the brackets are much weaker than the main ones.

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- Tomorrow: from Theory to Lab to Observations.
- Some examples.